UNIVERSITÉ DE CERGY-PONTOISE
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Thèse pour obtenir le grade de
Docteur en Sciences de Gestion de l'Université de Cergy-Pontoise

Finance immobilière
Essais sur la gestion de portefeuille et des risques

Une mesure du risque de l'immobilier direct

Version finale

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Real Estate Finance
Essays in Portfolio and Risk Management

A valuation of risk for direct real estate

Final version

by
Charles-Olivier AMÉDÉE-MANESME
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Avertissements / Warnings

Wisdom means to have sufficiently big dreams so as not to
lose sight of them while pursuing them.

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La Sagesse, c'est d'avoir des rêves suffisamment grands pour
ne pas les perdre de vue lorsqu'on les poursuit.

O. Wilde

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Summary

The academic contribution of this thesis is in providing an estimate of the risk for managing commercial real estate investment. Property investment is subject to numerous specificities including location, liquidity, investment size or obsolescence, requiring active management. These particularities make traditional approaches to risk measurement difficult to apply. We present our work in the form of four papers on real estate portfolios and risk management. This research is built on extant literature, and relies on previous research, examining first the implication of the option of the tenant to vacate embedded in leases and the implication of this for portfolio value, risk and management. The thesis then concentrates on valuation of Value at Risk measurements through two new approaches developed especially for real estate.

In the first paper, we consider options to vacate embedded in continental Europe leases in order to better assess commercial real estate portfolio value and risk, conducted through Monte Carlo simulations and options theory. The second paper considers the optimal holding period of a real estate portfolio when options to break the lease are considered. It relies directly from the first article, which has already treated this kind of option. The third paper proposes a model to determine the Value at Risk of commercial real estate investments, considering non-normality of real estate returns. This is conducted through a Cornish-Fisher expansion and rearrangement procedure. In the fourth paper, we present a model developed for real estate Value at Risk valuation. This model accounts for the most important parameters and specifications influencing property risk and returns.

Keywords: real estate, lease structure, portfolio management, risk management, Value at Risk, Cornish-Fisher expansion, Monte Carlo simulation, rearrangement procedure.

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Résumé

Cette thèse contribue à la recherche académique en immobilier par l'apport d'une estimation du risque pour la gestion d'immobilier commercial d'investissement. L'investissement immobilier compte de nombreuses particularités parmi lesquelles la localisation, la liquidité, la taille d'investissement ou l'obsolescence et requiert une gestion active. Ces spécificités rendent les approches traditionnelles de mesure du risque difficile à appliquer. Ce travail de recherche se présente sous la forme de quatre articles académiques traitant de la gestion de portefeuille et du risque en immobilier. Ce travail est construit sur la littérature académique existante et s'appuie sur les publications antérieures. Il s'attache d'abord à analyser les options de départ des locataires contenues dans les baux commerciaux en Europe continental et en étudie les impacts sur la valeur, la gestion et le risque des portefeuilles. Ensuite, la thèse étudie l'évaluation d'un outil de mesure du risque en finance, la *Value at Risk* au travers de deux approches innovantes spécialement développées pour l'immobilier.

Dans le premier article, nous prenons en considérations les options de départ des locataires inclus dans les baux en Europe continental pour mieux évaluer la valeur et le risque d'un portefeuille de biens d'immobilier commercial. Ceci est obtenu par l'utilisation simultanée de simulations de Monte Carlo et de la théorie des options. Le second article traite de la durée de détention optimale d'un portefeuille immobilier lorsque sont prises en compte les options contenues dans les baux. Le troisième article s'intéresse à la *Value at Risk* et propose un modèle qui tient compte de la non-normalité des rendements en immobilier. Ceci est obtenu par la combinaison de l'utilisation du développement de Cornish-Fisher et de procédures de réarrangement. Enfin dans un dernier article, nous présentons un modèle spécialement développé pour le calcul de *Value at Risk* en immobilier. Ce modèle présente l'avantage de prendre en compte les spécificités de l'immobilier et les paramètres qui ont une forte influence sur la valeur des actifs.

Mots clefs : immobilier, structure des baux, gestion de portefeuilles, gestion du risque, *Value at Risk*, développement de Cornish-Fisher, méthodes numériques, procédures de réarrangement.

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Résumé long de thèse

Cette thèse se place dans le cadre de la recherche en finance immobilière. Elle cherche en particulier à mieux comprendre, appréhender et mesurer les risques liés spécifiquement au secteur immobilier et plus particulièrement à l'immobilier d'investissement : l'immobilier commercial (*commercial real estate*). Ce travail a été réalisé dans le cadre d'un contrat *CIFRE* avec BNP Paribas Real Estate.

La thèse se divise en trois parties liées par le fil commun de la mesure du risque immobilier. Chaque partie compte deux chapitres ce qui fait un total de 6 chapitres. La thèse est rédigée en anglais et présente 4 articles de recherche originaux. Le premier chapitre présente le marché immobilier, son fonctionnement et les principaux véhicules d'investissement. Le second chapitre est une revue de la littérature. Cette revue de la littérature n'a pas prétention à être exhaustive mais est suffisamment large pour couvrir une très grande partie des sujets traités ou abordés dans cette thèse. Le but est d'exposer les principaux résultats de la recherche en finance immobilière. La seconde partie s'intéresse à la mesure et à la prise en compte du risque en immobilier (risque de marché et risque spécifique). Cette partie se concentre particulièrement sur le risque lié aux baux. Le troisième chapitre présente le premier article qui traite de l'évaluation immobilière et de l'analyse des risques par la combinaison de simulations de Monte Carlo et l'introduction de la théorie des options. Le quatrième chapitre offre une application directe du premier article. Il étudie la durée de détention optimale d'un portefeuille immobilier en fonction des baux des actifs immobiliers. Ce quatrième chapitre constitue le second article de cette thèse. La troisième partie se concentre précisément sur la Value at Risk en tant que mesure du risque financier. Cette mesure a été choisie par un certain nombre de règlementations récentes[1] pour le calcul du capital requis des banques et assureurs européens. Le cinquième chapitre propose un modèle de calcul de la Value at Risk (VaR) en immobilier prenant en compte la non-normalité des rendements de l'immobilier commercial. Ce modèle repose sur le développement de Cornish Fisher et

[1] Solvency II, réglementation qui concerne les assureurs européens, est en cours de mise en place à l'heure ou cette thèse est écrite. Cette règlementation a choisi la VaR à 0.5% pour le calcul du capital requis et conservé par les assureurs.

Bâle II, réglementation qui concerne les banques dans le monde entier est en vigueur depuis 2007. Cette réglementation base sur la VaR à 1% le calcul des risques de marché.

Bâle III est la révision de la norme Bâle II. Principalement cette réforme touche à la liquidité. La réforme propose en outre l'utilisation de la Value at Risk Conditionnelle (ou Expected Shortfall) pour le calcul du capital requis. A l'heure ou cette thèse est écrite, la réglementation n'est pas publiée et le débat sur le choix de la mesure la plus adaptée n'est pas arrêté.

une procédure de réarrangement. Il a fait l'objet de la rédaction d'un article. Le sixième et dernier chapitre expose un modèle de calcul de la VaR qui prend en compte les principaux risques spécifiques liés à l'immobilier et qui de suite permet un calcul de la VaR qui tient compte des spécificités du portefeuille. Ce modèle présente l'avantage de différencier les portefeuilles immobiliers sur le critère de la VaR.

L'immobilier requiert des connaissances multidisciplinaires alliant entres autres l'architecture, la construction, l'urbanisme, l'économie ou la finance. Ainsi, l'immobilier rentre dans de nombreuses disciplines académiques et dans de nombreux pays, l'immobilier fait l'objet de chaires spécifiques. Dans cette présentation, on va d'abord s'attacher à définir l'immobilier, puis on présentera succinctement le marché de l'immobilier et en particulier l'immobilier en tant que classe d'actifs et enfin, on s'attachera à donner certains des principaux canaux d'investissement en immobilier.

Dans son sens large, l'immobilier concerne tout ce qui touche à la Terre. Sur cette base, l'immobilier représente un quart de la surface de la planète. Dans une définition plus restreinte, l'immobilier représente l'ensemble des espaces construits ou exploitables de la planète. En ce sens, l'immobilier recouvre un certain nombre de sous-actifs qui ont des propriétés, classifications et caractéristiques différentes. Les principaux sous-actifs sont : les terrains, les bureaux, les propriétés résidentielles, les commerces, les espaces industriels, ou encore les hôtels. La plupart des acteurs se sont spécialisés dans un des sous-actifs avec une palette de métiers eux-mêmes spécialisés tels que la promotion immobilière, le développement, la construction, la gestion, l'investissement etc. L'immobilier est donc tout ce qui est lié au terrain, son développement, sa construction, sa vente, son achat et sa gestion.

Le marché de l'immobilier est un marché local. On distingue deux marchés : le marché de l'immobilier résidentiel (*housing*) et le marché de l'immobilier commercial. Dans cette thèse, on ne s'est intéressé qu'à l'immobilier commercial et même plus précisément à l'immobilier commercial d'investissement[2], soit les actifs achetés dans le but d'en retirer un rendement. En tant que classe d'actif, l'immobilier est une classe distincte des autres. Elle représente 50% de la richesse mondiale (source : *The Economist*), c'est la première classe d'actifs des investisseurs individuels, la plus vieille classe d'actifs, et elle est souvent présentée comme une classe d'actifs qui devrait être incluse dans tous les portefeuilles diversifiés. Cette classe d'actifs montre aussi des spécificités uniques : illiquidité, localisation, taille d'investissement ou encore

[2] La terminologie anglo-saxonne est plus précise sur le sujet car elle différencie l'immobilier détenu par les entreprises, *corporate real estate*, et l'immobilier utilisé par les entreprises, *commercial real estate*.

l'obsolescence. Depuis le début des années 90, le secteur de l'immobilier s'est financiarisé dans le monde entier créant de fait la finance immobilière. En Europe, on compte aujourd'hui une taille du marché de l'immobilier commercial presque comparable à celle des marchés obligataires ou actions (5 000 Mds€ versus 7 000 Mds€). Comme dans tous les domaines financiers, l'objectif est d'évaluer les principaux facteurs de risques et de rendements : l'état du marché immobilier locatif, l'état du marché immobilier d'investissement, les coûts opérationnels et de maintenance, les possibles coûts de vacance, la liquidité, les baux, les problématiques de financement etc. L'investissement dans cette classe d'actifs attire généralement des investisseurs en quête de diversification et de rendement récurrent mais aussi des investisseurs opportunistes recherchant des gains en capitaux. Comme annoncé précédemment, le marché immobilier est un marché local mais à capitaux internationaux. Le nombre d'investisseurs internationaux et de transactions transfrontalières croit fortement depuis le début des années 2000. Les acteurs du marché expliquent que cette augmentation est largement due à la transparence du marché qui s'est nettement améliorée depuis les années 90, en particulier avec l'émergence de fournisseurs de données spécialisés en immobilier. Cette internationalisation s'est aussi accompagnée d'une augmentation de la corrélation entre les marchés.

L'investissement en immobilier peut revêtir deux grandes formes : l'investissement direct ou indirect. Dans le cas de l'investissement direct, l'immobilier est détenu physiquement par les investisseurs. Ils sont donc en charge de gérer leurs actifs (location, travaux, mise aux normes etc.), éventuellement par un contrat de prestations de service. Dans le cas de l'investissement indirect, l'immobilier est détenu par le biais de véhicules d'investissement. Ces véhicules peuvent être cotés (foncières, *REITs*) ou pas ; règlementé (OPCI, fonds ouverts allemands) ou pas (fonds luxembourgeois). L'avantage de l'indirect est d'obtenir plus rapidement une diversification et de pouvoir profiter des compétences de spécialistes de l'investissement et de la gestion immobilière. Son inconvénient est son coût (frais de gestion).

La littérature en immobilier n'est relativement pas très large en comparaison avec d'autres classes d'actifs. Principalement, cette littérature est anglo-saxonne, en grande partie pour des questions d'accessibilité aux données. Elle est construite autour de quatre grandes thématiques : la gestion de portefeuille, les baux, la distribution des rendements et la Value at Risk. Sans prétendre être exhaustive, notre revue de la littérature aborde les principaux articles et souligne les résultats les plus intéressants relatifs à la thèse.

Dans le cadre de la gestion de portefeuille, la littérature en finance immobilière étudie différents aspects tel que l'allocation optimale en actifs immobiliers, la diversification d'un portefeuille immobilier par régions ou par type d'actifs, le nombre d'actifs ou de baux nécessaires à la diversification d'un portefeuille immobilier, la couverture contre l'inflation, le niveau optimal de dette ou encore l'étude du marché naissant des dérivés immobiliers. La littérature forme un consensus sur l'apport de l'immobilier dans un portefeuille. Tout portefeuille diversifié devrait posséder de l'immobilier. Les proportions varient selon les études et les pays entre 15 et 25% (Chaudhry et al., 1999 ; Hoesli et al., 2004 ; Bekkers et al., 2009 ; MacKinnon et Zaman, 2009). Cependant, les investisseurs institutionnels allouent une part beaucoup plus faible que celle recommandée par la littérature (Chun et al., 2004 ; Geltner et al. 2006 ; Clayton, 2007) : entre 5 et 10%. De nombreuses études se sont alors interrogées sur les raisons de cette faible part allouée à l'immobilier en contradiction avec les recommandations des modèles d'allocations. Les principaux résultats et articles sur le sujet sont ceux de Chun et al. (2004), Geltner et al. (2006) ou Bond et al. (2008). Il n'y a pas de consensus parmi les académiques pour expliquer la faible allocation des institutionnels en immobilier. Cependant, la littérature s'accorde sur la difficulté rencontrée par les investisseurs pour évaluer le risque en immobilier. En effet, un certain nombre de particularités propres à l'immobilier rendent cette classe d'actifs plus singulière que les classes d'actifs traditionnelles (actions ou obligations). Deux effets sont spécifiques à l'immobilier : d'abord la diversification d'un portefeuille d'actifs immobiliers requiert un nombre très important d'actifs (> 200) car le risque spécifique est très difficile à éliminer. En effet, la littérature montre que la diminution marginale du risque est quasiment nulle après 10 actifs en immobilier (Brown, 1991 ; Byrne et Lee, 2001, Callender et al., 2007). Ensuite la gestion immobilière requiert une gestion active, le gérant étant responsable de l'exécution des transactions, de la sélection des actifs et de la gestion du risque mais aussi de l'exécution de la stratégie qu'il souhaite mettre en place pour chacun des actifs. Contrairement aux gérants actions ou obligations qui sélectionne des entreprises (ou des pays) et qui gère les probabilités qu'elles n'exécutent pas les stratégies qu'elles annoncent, le gérant immobilier doit comprendre les fondamentaux du marché et être capable d'évaluer les risques de chacun des actifs et au niveau du portefeuille, pour décider et exécuter une stratégie. Le gérant immobilier doit donc être en mesure de suivre la vie de l'actif et d'en mesurer les risques à chaque instant. C'est en ce sens que cette thèse a été écrite. Cette thèse se concentre sur la compréhension des spécificités de l'immobilier et en particulier sur la mesure du risque en immobilier lorsque ses particularités sont prises en considération. En effet, il n'existe pas de modèles spécifiques d'évaluation du risque en immobilier ou qui prend en compte les risques de l'immobilier. C'est en partie pour ces raisons que le marché de l'immobilier n'a jamais vu se développer réellement dans le marché de produits dérivés (c'est pourtant aussi un champ de recherches académiques large). Deux autres domaines de recherche ont fait l'objet d'un fort intérêt de la part de la communauté des chercheurs en immobilier : il s'agit de la couverture (supposée) contre l'inflation et du niveau optimal de dette dans un portefeuille immobilier. Ces deux domaines ont fait l'objet de papiers de recherche qui ne font pas partie de cette thèse.

Le bail est un élément essentiel de l'immobilier. Le bail est un contrat de location lié à l'actif immobilier. Il est essentiel en finance immobilière car il est à la base des échanges de flux. Outre la définition juridique de jouissance d'une chose immobilière pour une durée donnée, le contrat de bail stipule le montant du loyer, son mode d'ajustement, la durée du bail et les éventuelles conditions de rupture. Les baux ont fait l'objet d'un grand nombre de recherches dans la littérature en finance immobilière mais aussi en économie immobilière. De nombreux articles s'intéressent à la valorisation financière des baux et à l'influence qu'ils ont sur la structure des loyers. Dans cette thèse, nous nous intéressons particulièrement à l'influence de la structure des baux sur le risque immobilier et sur l'évaluation immobilière. En effet les baux commerciaux (européens en particulier) sont en général signés sur des durées longues avec des options de départ anticipées possibles en faveur du locataire en cours de contrat à des dates données. On s'intéresse dans cette thèse à l'analogie qu'il y a entre les options financières et les options de départ des locataires dans les baux commerciaux.

La distribution des rendements immobiliers est un sujet récurrent dans la littérature en finance immobilière. La littérature sur le sujet est principalement anglo-saxonne et fait l'objet d'un consensus : les rendements immobiliers ne suivent pas une distribution normale. Les articles sur le sujet sont basés soit sur l'immobilier direct, soit sur l'immobilier coté. Les travaux de Young (1995, 2006, 2008) sont une référence dans le domaine. Dans cette thèse, on s'intéresse à l'immobilier direct et la non-normalité des rendements nous amène à utiliser des techniques qui prennent en compte cette non-normalité pour déterminer la Value at Risk.

La Value at Risk est une mesure de risque relativement récente (années 90) qui a connu un fort essor à la lumière des diverses régulations qui se sont imposées aux acteurs de la finance. L'objet de la thèse n'est pas de discuter la pertinence de la VaR comme mesure de risque ou de dénoncer ses limites. Les régulateurs de nombreux pays (entre autres, ceux concernés par Bâle II, Bâle III et Solvency II) ont choisi la VaR comme mesure de risque (pour le calcul entre autre du capital requis) et de fait, s'intéresser à la VaR est essentiel même s'il faut rester conscient de ses limites. La VaR a fait l'objet de travaux très nombreux. Les travaux fondateurs sur la mesure de la VaR sont, entre autres, ceux de Jorion (1996), Linsmeier et Pearson (2000), Duffie et Pan (1997) ou Engle et Manganelli (1999). Les propriétés théoriques ont été abordées par Artzner et al.

(1999), Cvitanic et Karatzas (1999) ou encore Wang (1999). L'article d'Artzner et al. (1999) est essentiel dans la littérature. De nombreux articles se sont aussi intéressés à la meilleure méthode pour calculer la VaR, entre autres, ceux de Pichler et Selitsch (1999) et Mina et Ulmer (1999) s'intéressent à la décomposition de Cornish Fisher pour le calcul de la VaR. La littérature relative à la VaR spécifique à l'immobilier est pratiquement inexistante. L'immobilier indirect coté n'a pas fait l'objet de recherches spécifiques car les outils et méthodologies que l'on peut appliquer sont ceux qui ont été développés pour les autres classes d'actifs. Pour l'immobilier direct, la littérature est pratiquement inexistante. Pourtant le besoin de méthodes et outils spécifiques se fait fortement ressentir. Ceci est souligné dans le rapport pour IPF (*Investment Property Forum*) écrit par Booth et al. (2002) et qui revoit l'ensemble des méthodes de mesure et de gestion du risque. Le seul article spécifique sur la VaR est l'article de Gordon et Wai Kuen Tse (2003) qui considère la Value at Risk comme une mesure de risque pour prendre en compte l'effet de levier. C'est en particulier cette absence de recherche sur la VaR en immobilier qui a motivé le travail de thèse sur la VaR immobilière.

Le troisième chapitre (premier article)[3] *Combining Monte Carlo Simulations and Options to manage Risk of Real Estate Portfolio* se concentre sur l'analogie qui existe entre les baux en immobilier commercial et les options financières. Un bail donne généralement au locataire le droit mais pas l'obligation de partir avant la fin du contrat à des échéances données (traditionnellement un bail est signé pour une durée donnée avec une ou plusieurs options de départ en faveur du locataire au cours de la durée du bail). De la même façon, une option européenne donne le droit mais pas l'obligation à son détenteur de vendre ou acheter un sous-jacent financier à une date donnée. Si l'on fait l'hypothèse que les acteurs sont rationnels, ces options ne seront exercées que si elles sont dites « dans la monnaie ». Par analogie, on peut envisager que sous l'hypothèse d'un comportement rationnel des acteurs, une option de départ en faveur d'un locataire sera exercée si la valeur des loyers de marché est inférieure au loyer payé actuellement (le loyer payé devenant la valeur du strike et l'option de départ étant « dans le monnaie » dans le cas où le locataire rationnel doit l'exercer). C'est sur la base de cette analogie que ce chapitre est construit. De la même façon qu'une option financière est exercée, le chapitre

[3] Ce chapitre a fait l'objet d'un article de recherche accepté pour publication dans le *Journal of Property Investment and Finance*. Cet article a été coécrit avec Michel Baroni, Fabrice Barthélemy et Etienne Dupuy. L'article a fait l'objet de présentations dans de nombreuses conférences telles que celle de l'American Real Estate Society en 2010, l'European Real Estate Society en 2010, l'American Real Estate Urban Economic Association (American Economic Association) en 2012 et l'AFFI en 2012.

intègre les options de départs des locataires dans un modèle d'évaluation qui prend ainsi en compte les risques liés aux baux. Ces options de départ contenues dans les baux sont l'une des principales préoccupations des investisseurs. En effet, les investisseurs en immobilier sont majoritairement attirés par deux choses : d'une part les flux récurrents et indexés que procurent l'immobilier et d'autre part les potentielles plus-values immobilières liées à la corrélation entre cet actif et le niveau d'inflation. Cependant, les options contenues dans les baux ont une grande influence sur la récurrence des flux mais aussi sur la valeur des actifs[4]. C'est pourquoi il est fondamental de les prendre en considération lors de l'évaluation d'un actif et dans le cadre de la gestion de portefeuilles immobiliers. En effet, les modèles traditionnels de gestion et d'évaluation prennent mal en compte cette spécificité immobilière : soit un revenu moyen récurrent est considéré, soit deux ou trois (en général : cas de base, optimiste ou pessimiste) sont pris en compte auxquels on affecte éventuellement une probabilité d'occurrence.

La nécessité d'une approche qui tient compte des risques spécifiques provient d'une part de la mauvaise appréciation du risque lié aux baux et d'autre part des difficultés à faire disparaitre le risque spécifique des portefeuilles immobiliers. En effet, comme présenté dans la revue de la littérature, un portefeuille immobilier diversifié nécessite un très grand nombre d'actifs. Ce nombre d'actifs est rarement atteint par les investisseurs. Par suite, le risque spécifique demeure dans le portefeuille et il est donc nécessaire de le prendre en compte dans les modèles d'évaluation. C'est là l'idée de base de ce premier chapitre. C'est justement de proposer un modèle qui pallie aux défauts des modèles plus traditionnels importés de la finance. Notre approche suggère de combiner l'utilisation de méthodes numériques (Monte Carlo) et de la théorie des options (cependant l'objet n'est pas de valoriser la prime d'option mais seulement d'utiliser la théorie des options : exercice ou pas). L'idée est d'utiliser des simulations de Monte Carlo pour les valeurs locatives de marché et pour le prix du portefeuille (en prenant en compte la corrélation entre les différents facteurs de risques estimés) puis, aux dates déterminées, de comparer le loyer payé avec le loyer disponible (simulé) sur le marché pour un bien identique et considérer la décision la plus rationnelle du locataire. Ainsi, si face à une option de départ, si le loyer payé est supérieur (à la constante [5] près) au loyer de marché, le locataire quitte l'immeuble et le propriétaire fait face à une période de vacance et donc à un vide dans ses revenus. Eventuellement, selon les situations et les marchés, cette période peut générer des coûts de vacance. La durée de la vacance est modélisée par une

[4] Particulièrement en immobilier commercial où un bien loué se vend plus cher et plus vite qu'un bien vacant par opposition au marché résidentiel.

[5] La constante peut être interprétée comme des coûts de transactions, de déménagements ou de frictions.

loi de Poisson. Dans notre cas et pour simplifier la présentation, nous prenons l'hypothèse qu'à la fin d'un bail, les deux acteurs ayant une option (de départ ou de reprendre son bien), ils négocient un nouveau bail à la valeur locative de marché. La figure 1 présente le cas sur un bail type français, soit un bail de 9 ans avec deux possibilités de départ pour le locataire en année 3 et 6 (dit le bail 3/6/9). Cette figure illustre le cas ou à la fin de l'année 3, le loyer de marché (*MRV*) est inférieur au loyer payé et le locataire quitte l'immeuble. Le propriétaire fait face à 4 années de vacance et un nouveau bail est contracté jusqu'à la fin de la simulation.

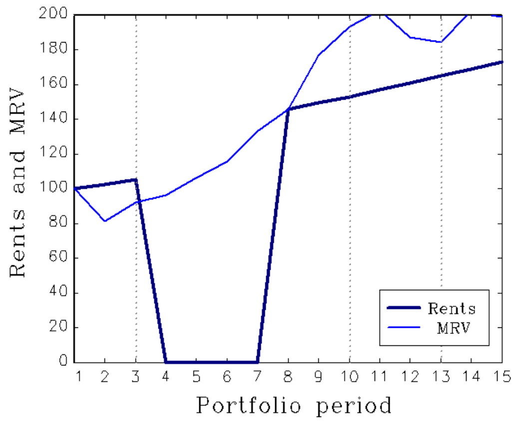

Figure 1 - Illustration du modèle sur un bail type français (3/6/9) avec = 0 pour 1 simulation

Ensuite, cette action est répétée de très nombreuses fois et on peut obtenir la moyenne des flux reçus sur un bail. Le résultat est présenté dans la figure 2. On observe une forte baisse des loyers générés par le bail type aux années ou le locataire à une possibilité de départ.

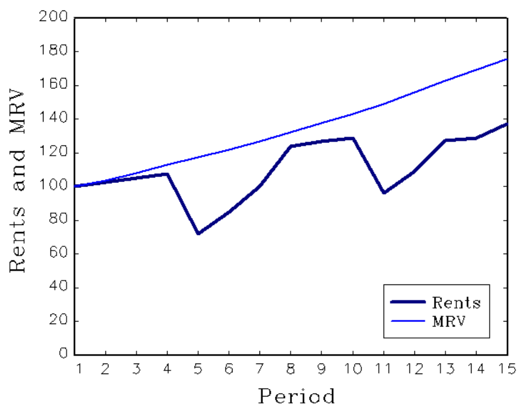

Figure 2 - Moyenne des loyers générés et des valeurs locatives de marché d'un bail type français (3/6/9) avec = 0 pour 10 000 simulations

L'intérêt de cette méthode en plus d'une meilleure prise en compte des risques spécifiques dans la valorisation d'un portefeuille immobilier est aussi de mieux appréhender et évaluer les risques immobiliers. En effet, l'utilisation des méthodes numériques permet d'évaluer avec plus de pertinence les risques du portefeuille, en particulier, la distribution des loyers ou la distribution des valeurs possibles de portefeuilles permet une meilleure prise en considération des risques. La figure 3 illustre ce phénomène. On observe qu'à la période 8, la distribution est fortement centrée sur deux points, ce qui laisse présager un grand nombre de possibilités de départ à cette date. De plus l'obtention de distribution donne la possibilité de déterminer la VaR de l'investissement ou d'autres mesures de risque lié à la distribution.

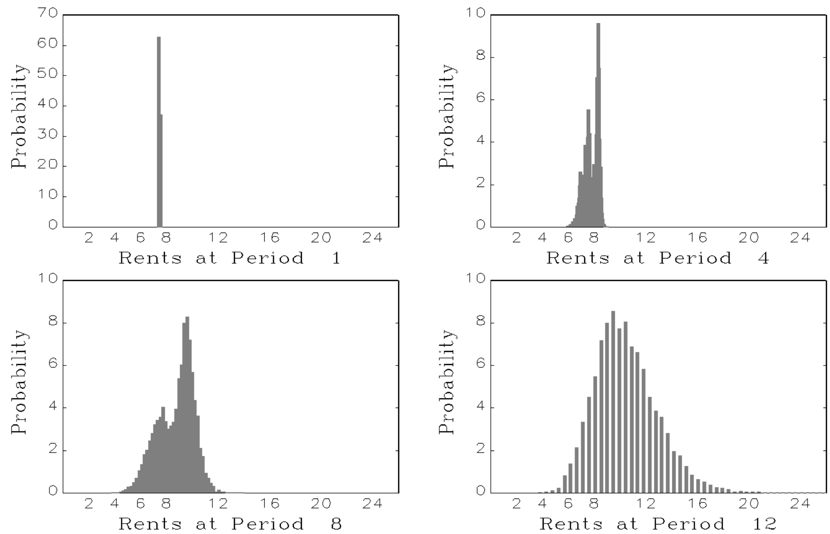

Figure 3 - Distribution des loyers générés par un portefeuille aux années 1, 4, 8 et 12

Dans ce chapitre, nous avons proposé une nouvelle méthode pour l'évaluation d'un portefeuille immobilier qui utilise des simulations de Monte Carlo et la théorie des options pour le calcul des valeurs du portefeuille. Ceci a permis d'intégrer la structure des baux dans le processus d'évaluation. Le modèle prend ainsi en compte le comportement des locataires et l'influence de ce comportement sur les flux de trésorerie. Une loi de Poisson est utilisée pour déterminer la durée de la vacance (comptage des périodes vacantes). Du point de vue du praticien, le modèle peut être utilisé pour calculer des évaluations de portefeuilles ou d'actifs plus pertinentes. Avant tout, l'intérêt réside dans la possibilité conférée par le modèle d'obtenir un histogramme. L'approche est flexible et permet de rajouter et de modifier de nombreux paramètres en fonction des besoins inhérents à chaque marché et à chaque investisseur.

Ce travail ouvre la voie à de nombreux autres domaines de la finance immobilière, la gestion des risques en particulier. La connaissance des flux de trésorerie est une aide précieuse pour la mesure des risques et dans les négociations entre propriétaire et locataire. En utilisant des méthodes de Monte Carlo, on obtient aussi un ensemble de résultats au lieu d'un résultat unique ce qui présente un intérêt évident pour la gestion des risques car les régulateurs comme les investisseurs ont de plus en plus besoin de mesures de risque. Le développement de notre démarche va dans ce sens.

Le quatrième chapitre (second article)[6] de cette thèse est une application directe du modèle précédent. L'article présenté dans ce chapitre se concentre sur un problème traditionnel de la Finance : la durée de détention. La littérature sur le sujet a créé un consensus : des coûts de transaction élevés impliquent une durée de détention plus longue et une forte volatilité implique une durée de détention plus courte. A ce sujet, on peut se reporter aux travaux de Demsetz (1968), Tinic (1972), Amihud et Mendelson (1986) ou encore Atkin et Dyl (1997). L'immobilier qui présente une forte volatilité et des coûts de transactions élevés est un cas à part sur lequel la littérature n'a pas su trouver un consensus. De plus, le caractère local et les spécificités pays de l'immobilier créent de grandes différences. Par suite, il convient de considérer les spécificités de l'immobilier pour déterminer la durée de détention optimale. Dans ce contexte, le second article se propose de prendre en compte les baux inclus dans le portefeuille pour déterminer la durée de détention optimale du portefeuille. Ceci est rendu possible grâce à l'utilisation du modèle développé précédemment[7].

Ce travail fait suite à un travail précédent publié par Baroni et al. (2007b) et qui donne une formule fermée qui permet d'obtenir la durée de détention optimale d'un portefeuille immobilier par l'utilisation de méthodes de simulations de Monte Carlo. Notre objectif est d'utiliser une méthodologie proche mais pas similaire. L'idée est de

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[6] Cet article a fait l'objet d'un article présenté à la conférence annuelle de l'American Real Estate Society en 2012. Sa rédaction a été finalisée avec l'écriture de cette thèse et il est maintenant en révision au *Journal of Property Investment and Finance.*

[7] Peu de recherches examinent les périodes de détention optimales pour les portefeuilles immobiliers. Toutefois, récemment, ce sujet a fait l'objet de quelques publications : Barthélémy et Prigent (2009, 2011) ou Cheng et al (2010a, 2010b, 2010c). A ce jour et à notre connaissance, aucune recherche ne prend en compte la structure des baux dans l'évaluation de la durée de détention optimale. Cet article cherche à combler cette lacune.

rajouter la structure des baux et donc de prendre en compte les options incluses dans les baux (au lieu d'un loyer moyen tel que utilisé dans Baroni et al., 2007b). Ces options modifient la distribution des valeurs et par suite la durée de détention optimale. Ce chapitre démontre les différences qui se produisent lorsque les options de départ accordées aux locataires sont prises en considération. Nous démontrons comment les objectifs de détention peuvent être modifiés par la prise en considération de la structure des baux du portefeuille. L'objectif n'est pas de prédire la période de détention optimale, mais d'analyser l'effet des paramètres sur la durée de détention optimale. Les résultats sont illustrés sur les deux figures 4 et 5.

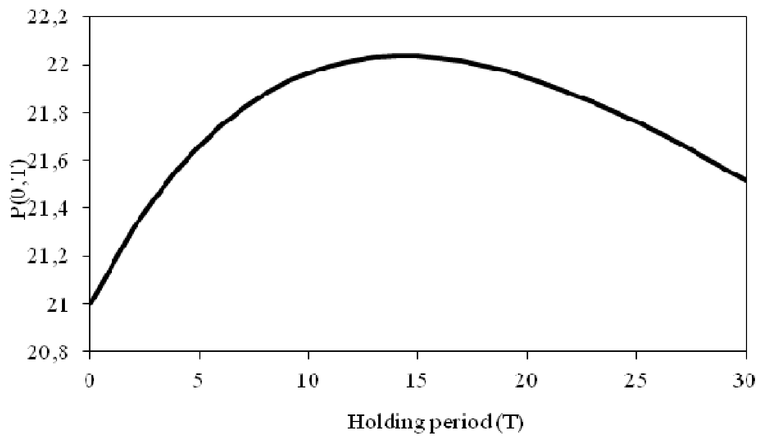


Figure 4 - Durée de détention optimale d'un portefeuille lorsque seule la valeur terminale est simulée (cas Baroni et al, 2007b)

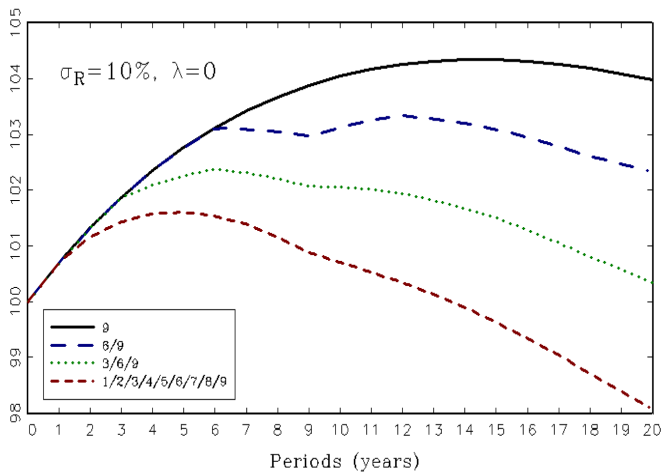


Figure 5 - Durée de détention optimale d'un portefeuille avec simulations de la valeur terminale, des valeurs locatives de marché, et de la structure des baux (ici : bail 9 ans ferme ; 6/9 ; 3/6/9 ; 1/2/3/4/5/6/7/8/9)

Dans ce chapitre, nous avons donc proposé de tenir compte de la structure des baux dans le calcul de la durée de détention optimale d'un portefeuille de biens immobiliers en utilisant le modèle proposé au chapitre précédent. En grande partie, le meilleur moment pour vendre un portefeuille immobilier dépend des flux futurs de trésorerie. Nos principaux résultats sont les suivants : d'abord la volatilité des valeurs locatives de marché a une très forte influence sur la période de détention (plus la volatilité augmente, plus la durée de détention diminue), ensuite, le nombre d'options a un effet très fort sur la durée de détention. En somme, prendre en compte la structure des baux et par suite les possibilités de rupture données aux locataires permet d'obtenir une meilleur évaluation et analyse de la durée de détention optimale.

Le chapitre 5 (troisième article)[8] traite de la mesure de la Value at Risk lorsque la non-normalité des rendements est prise en compte. Pour ce faire, ce chapitre propose l'utilisation du développement de Cornish-Fisher qui permet d'approcher les quantiles d'une distribution lorsque celle-ci ne satisfait pas l'hypothèse de normalité.

Comme présenté dans la revue de la littérature, la distribution des rendements en immobilier ne peut pas être décrite par une loi normale. La littérature sur le sujet est relativement large et crée un consensus. C'est pourquoi, on ne peut faire l'hypothèse d'une distribution Gaussienne pour le calcul de la VaR. Traditionnellement, cette hypothèse est acceptée car elle permet de calculer très rapidement la VaR avec comme seule information la moyenne et la variance. Ce chapitre propose de mesurer la VaR en utilisant les 4 premiers moments de la distribution (moyenne, variance, coefficient d'asymétrie et Kurtosis). Pour ce faire, le développement de Cornish-Fisher est introduit. Ce développement permet d'approximer le quantile d'une distribution.

Si l'on note z_α le quantile gaussien et $z_{CF,\alpha}$ le quantile de Cornish Fisher, le niveau de probabilité, S le coefficient d'asymétrie et K le Kurtosis, le développement de Cornish-Fisher prend la forme suivante :

$$\forall \alpha \in \left]0,1\right[,\ z_{CF,\alpha} \simeq z_\alpha + \frac{1}{6}\left(z_\alpha^2 - 1\right)S + \frac{1}{24}\left(z_\alpha^3 - 3z_\alpha\right)(K-3) - \frac{1}{36}\left(2z_\alpha^3 - 5z_\alpha\right)S^2$$

On peut ainsi en déduire le quantile modifié de Cornish-Fisher :

$$\forall \alpha \in \left]0,1\right[,\ q_{CF,\alpha} = \mu + z_{CF,\alpha}\sigma$$

[8] Ce chapitre a fait l'objet de la rédaction d'un papier de recherche qui a été présenté en 2012 à la conférence annuelle de l'European Real Estate Society dans la session doctorale. L'article a été plébiscité et a reçu le prix du papier de recherche le plus recommandé de la session doctorant (*Most Commended Paper*). L'article est en cours de soumission au *Journal of Real Estate Finance and Economics.*

Ce développement de Cornish-Fisher permet donc de calculer rapidement le quantile d'une fonction en prenant en compte les moments d'ordre supérieur à 2. Bien que cette approximation soit un outil utile et puissant, il est peu utilisé en finance. Ceci provient d'une des limites du développement de Cornish-Fisher, il ne conserve pas la monotonie, pourtant condition nécessaire pour les fonctions de répartition : l'ordre des quantiles de la distribution n'est pas conservé par la transformation. Le développement de Cornish-Fisher viole donc une des conditions de base des fonctions de répartition.[9] Une condition nécessaire et suffisante pour conserver la monotonie est que la dérivée de $z_{CF,\alpha}$ par rapport à z_α ne soit pas nulle. Cela peut se traduire par :

$$\frac{S^2}{9}-4\left(\frac{K-3}{8}-\frac{S^2}{6}\right)\left(1-\frac{K-3}{8}-\frac{5S^2}{36}\right)\leq 0$$

Il faut donc que S et K respecte les conditions permettant de satisfaire l'inéquation précédente.

En pratique, cette condition n'est que rarement vérifiée ce qui rend l'utilisation du développement de Cornish-Fisher compliquée. Cette difficulté a été résolue par Chernozuhov et al. (2010). Il propose d'introduire une procédure de réarrangement pour résoudre le problème de la non-monotonie. Le réarrangement consiste à classer par ordre croissant ou décroissant l'ensemble des éléments d'une base de données. Cet article, Chernozuhov et al. (2010), démontre que l'utilisation d'une procédure de réarrangement permet d'une part de résoudre le problème de la non-monotonie dans l'utilisation de l'approximation de Cornish Fisher et d'autre part d'obtenir une meilleure estimation des quantiles. La figure 6 illustre ce principe.

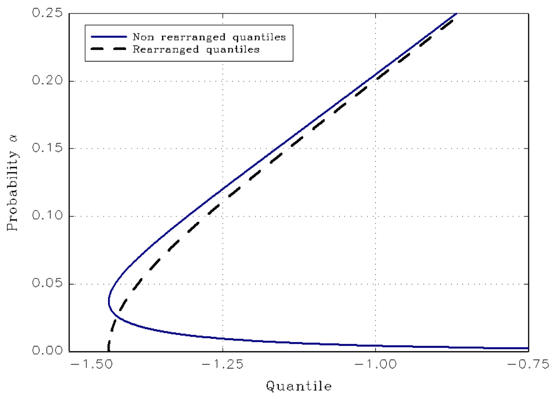


Figure 6 - Illustration de la procédure de réarrangement

[9] Ce point est largement débattu dans la littérature académique (Barton et Dennis, 1952 ; Draper et Tierney, 1971 et Spiring, 2011).

Dans ce chapitre, nous proposons d'estimer les quantiles des rendements immobiliers en utilisant cette combinaison (développement de Cornish Fisher et réarrangement) afin de déterminer la *Value at Risk* de l'immobilier lorsque les moments d'ordre supérieur à deux sont pris en compte. Nous appliquons cette méthodologie à l'indice IPD rendement en capital entre Janvier 1988 et Décembre 2010. En ce sens, nous nous plaçons très près du travail du régulateur pour Solvency II qui prend l'indice IPD rendement total (comprenant les revenus de loyers) sur les mêmes périodes. Nous choisissons l'indice rendement en capital car nous le considérons comme plus pertinent dans le calcul de la VaR qui traite de la perte de valeur. Le résultat est donné dans la figure 7. On peut observer que le cas Gaussien donne une VaR proche de 25% (valeur retenue par le régulateur pour l'immobilier avec une autre méthode et une autre base de données) et que lorsque les moments d'ordre supérieur à deux sont pris en compte, la VaR monte à 36%.

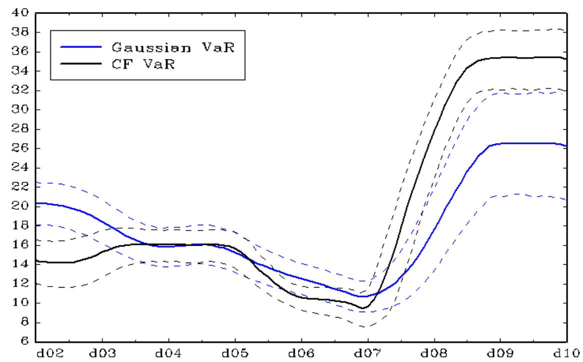


Figure 7 - Calcul de la Value at Risk à 0.5% sur une fenêtre glissante de 15 ans.

L'approche que nous proposons présente de nombreux avantages. Elle ne repose sur aucune hypothèse de distribution, le manque de données (problème classique de l'immobilier) est dépassé par cette approche (la méthode historique, par exemple, nécessite une grande quantité de données) et elle permet de prendre en compte les moments d'ordre supérieur à 2. Nos résultats suggèrent que les méthodes qui ne tiennent pas compte des moments d'ordre supérieur à 2 pour calculer la VaR donnent une mauvaise estimation du risque. En présence de rendements asymétriques et de queues épaisses, la VaR gaussienne conduit à des exigences de capital non adéquates et à une sous-évaluation de leur montant. Cette situation semble surtout apparaitre après la crise des subprimes. Ce chapitre a une pertinence particulière pour les praticiens du risque qui doivent calculer le capital requis dans un cadre règlementaire. Il ouvre en outre le champ à de très nombreuses autres recherches.

Le dernier chapitre (quatrième article)[10] de la thèse traite aussi de la Value at Risk. L'idée de base de ce chapitre repose sur une observation simple : les méthodes traditionnelles d'analyse et de mesure du risque ne permettent généralement pas de discriminer entre les stratégies ou entre les portefeuilles sur le critère de la Value at Risk. En effet, les méthodes traditionnelle, historique, Monte Carlo ou paramétrique sont basées sur des indices ou des bases de données qui concentrent et agrègent les informations de l'ensemble du marché. La VaR calculée avec ces méthodologies donne la VaR du marché ou à défaut la VaR correspondant aux données du marché (si l'ensemble du marché n'est pas observable) et par suite ces méthodologies ne donne pas la VaR du portefeuille ou de l'allocation spécifique de l'investisseur. Cette observation revêt une importance particulière en immobilier ou le risque de marché est difficilement isolable et les portefeuilles rarement correctement diversifiés, comme cela a été évoqué dans la revue de la littérature et dans l'introduction de la thèse ou de ce résumé. Pourtant deux portefeuilles d'actifs immobiliers qui ont des stratégies différentes ne devraient pas avoir la même VaR et par suite le même capital requis.

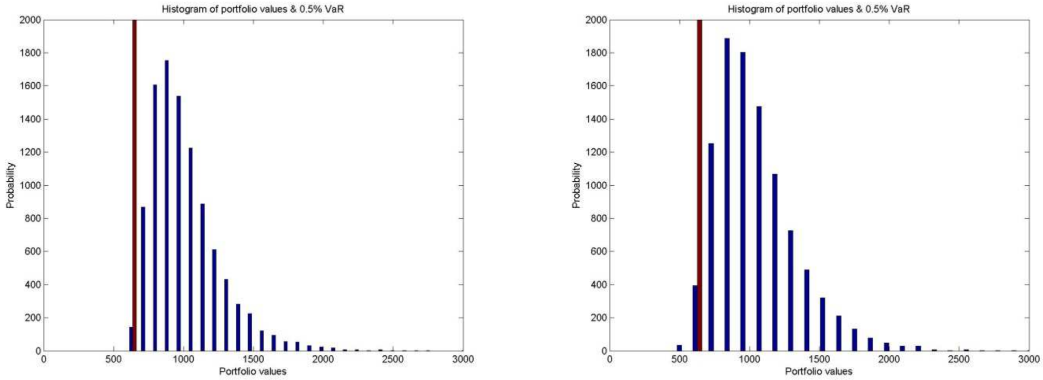
La méthodologie proposée repose sur des techniques de simulation et la prise en compte d'un certain nombre de spécificités de l'immobilier. Encore une fois, l'idée sous-jacente de ce chapitre repose sur une des particularités de l'immobilier, le risque spécifique est difficile à diversifier et il convient donc de considérer les risques qui proviennent des spécificités. Les risques spécifiques pris en compte dans cet article sont les structures de baux, l'obsolescence et son influence sur la durée de vacance entre deux locataires et sur la probabilité de vacance, le cout de la vacance et l'effet de levier. De plus, le risque de marché est pris en compte par des méthodes numériques.

Dans ce chapitre, la méthode numérique que nous retenons pour la prise en compte du risque de marché est le bootstrapping. Cette méthode présente l'avantage de ne pas requérir d'hypothèse sur la distribution des séries utilisées (mais on peut aussi utiliser des simulations de Monte Carlo si l'on peut aisément faire une hypothèse ou une estimation sur la distribution des rendements). La structure des baux est prise en compte par le modèle présenté dans le troisième chapitre de cette thèse (premier article) : si un bail est loué à un loyer différent du prix du marché, le locataire, rationnel, quitte le bâtiment et le propriétaire fait alors face à de la vacance. Il peut cependant exister une

[10] Cet article a été présenté à de nombreuses conférences. Entre autre, à la conférence annuelle de l'European Real Estate Society, en 2011 en session doctorale (poster) et en 2012 lors de la session plénière. Il a aussi été présenté lors de la session doctorale de la conférence annuelle de l'American Real Estate Society en 2012. En 2011, cet article a reçu le prix du meilleur poster de la session doctorale (*Best Poster Presentation*) à la conférence annuelle de l'European Real Estate Society.

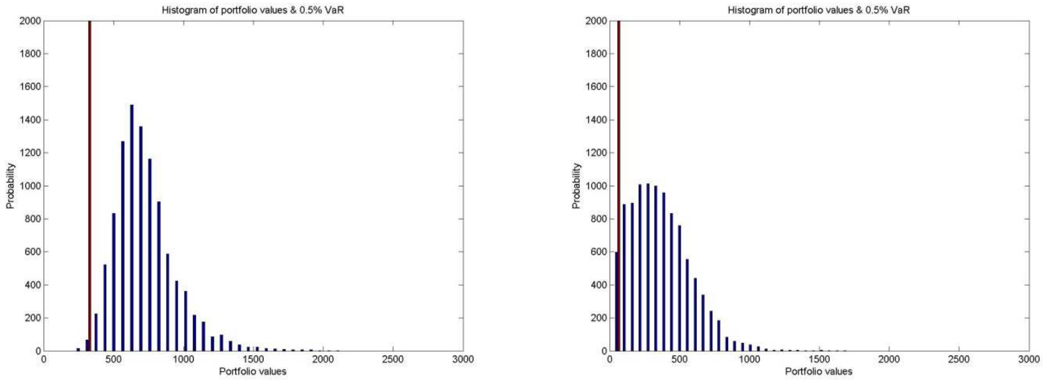
certaine latitude entre le prix du marché et le prix payé. La durée de la vacance est prise en compte par une loi de Poisson qui compte le nombre de périodes vacantes. Le loyer est donné par contrat et les valeurs locatives de marché sont simulées (bootstrapping). Si plusieurs marchés doivent être pris en compte, on utilise une méthode de moving block-bootstraping (la taille des blocs peut varier). L'obsolescence de l'actif est prise en compte par un taux d'obsolescence (ou d'usure) de l'actif immobilier. Ce taux influe sur le risque de devenir vacant comme sur la durée de la vacance. Enfin on considère l'effet de levier car c'est un des facteurs qui différencie les stratégies. On propose dans ce modèle de se baser sur le critère de la *loan to value* (LTV), avec une possibilité pour l'emprunteur de s'écarter un peu de la LTV initiale mais en considérant une rupture du contrat de prêt avec exercice de l'hypothèque sur le bien dans le cas où l'écart avec la valeur initiale est supérieur à un certain pourcentage négocié. Au regard des différences de stratégies possibles, on n'a pas considéré dans ce modèle le service de la dette (DSCR ou ICR) : ceci se justifie par le fait que les stratégies opportunistes peuvent ne pas générer de flux pendant un grand nombre de périodes et pourtant générer un fort rendement par des gains en capitaux d'où un rendement supérieur.

Le modèle est illustré sur 2 portefeuilles qui ont des stratégies radicalement différentes. Un portefeuille core (sécurisé) et un portefeuille opportuniste. Le portefeuille core investit dans des immeubles neufs ou récents, loués sur de longues périodes (baux fermes supérieurs à 6 ans) avec peu d'effet de levier (30%) et sur la base d'un taux de rendement initial proche de 6%. Le portefeuille opportuniste investit dans des actifs obsolètes ou anciens qui peuvent soit être loués sur des durées relativement courtes, soit être vacants avec un effet de levier conséquent (70%) et sur la base d'un taux de rendement initial de l'ordre de 8%. Les figures 8 et 9 présentent les résultats pour les deux portefeuilles, dans le cas d'abord (figure 8) où seuls sont pris en compte les risques de prix, de valeurs locatives et de baux, puis (figure 9) où sont ajoutés les risques liés à l'obsolescence, à la vacance et à la dette.



a. Core ($VaR_{0.5\%}$ = 34.7%) **b. Opportuniste ($VaR_{0.5\%}$ = 41.8%)**

Figure 8 - Cas où l'on considère le risque de prix et de valeur locative de marché et la structure des baux



a. Core ($VaR_{0.5\%}$ = 54.1%) **b. Opportuniste ($VaR_{0.5\%}$ = 94.4%)**

Figure 9 - Cas où l'on considère le risque de prix et de valeur locative de marché, la structure des baux, l'obsolescence et son influence sur la probabilité de devenir vacant et sur la durée de la vacance, le coût de la vacance et l'effet de levier

Le but de cet article est de contribuer à la littérature sur la VaR en proposant une méthode d'évaluation qui prenne en compte les spécificités de l'immobilier. Les modèles traditionnels de calcul de la VaR souffrent en immobilier d'une difficulté : ils s'appuient sur des indices de marché ou de données, et donc fournissent le même résultat indépendamment du portefeuille. Pourtant, les investissements immobiliers sont caractérisés par l'hétérogénéité des actifs et donc les indices boursiers ne reflètent généralement pas le risque du portefeuille de l'investisseur. Contrairement aux approches traditionnelles, notre approche permet de discriminer parmi les stratégies sur le critère de la VaR. Ceci peut être autant utile aux académiques qu'aux praticiens. Le modèle proposé ouvre la porte à de nombreuses études futures : l'impact de la dette, ou l'influence de la stratégie sur le capital requis. Le modèle est particulièrement pertinent, car il est le premier qui permet de calculer des VaR différentes pour différents portefeuilles, même si les portefeuilles ne diffèrent que sur leurs stratégies en termes de durée des baux.

*

*   *

Finalement, cette thèse a abordé la problématique du risque dans les investissements immobiliers et en particulier des spécificités de l'actif immobilier. Ce travail a grandement été motivé par d'une part le fait que la réduction marginale du risque idiosyncrasique diminue rapidement après 10 actifs et d'autre part par le fait que le risque spécifique est très difficile à diversifier, même en présence d'un portefeuille très large. Il est donc nécessaire de prendre en considération ces risques dans l'évaluation, la gestion et l'allocation de portefeuilles immobiliers. Le manque de travaux universitaires dans le domaine de l'immobilier direct et la taille de cette classe d'actifs dans le monde ont aussi été de grandes motivations dans l'écriture de cette thèse.

A plusieurs égards, la thèse répond à la problématique du risque spécifique en immobilier : elle propose de nouveaux modèles qui prennent en compte les caractéristiques spécifiques de l'investissement immobilier en termes de particularités comme dans les distributions de rendement. L'originalité réside en partie dans ces spécificités prises en compte et en partie dans l'approche pratique des méthodes proposées. Ce travail a un intérêt à la fois pour les praticiens et les académiques. Ces nombreuses applications pratiques ont été permises grâce au contrat doctoral (CIFRE) dans lequel cette thèse a été écrite.

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I started this thesis to expand my knowledge of real estate, particularly in Finance. With hindsight, I can say these 3 years at the University of Cergy-Pontoise not only expanded my knowledge of real estate but also expanded my horizon of life. Overall, the experience was gratifying in terms of insights gained and skills developed.

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[11] French real estate research association

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Never least, but too often last in my focus are both my family and friends for supporting me. They suffered for well over 4 years my grumpiness and impatience.

Finally, I thank everybody who was important to the realization of this thesis, and express my apologies that I could not mention him or her personally.

My apologies to anyone I missed inadvertently. I assume responsibility for any remaining errors.

Charles-Olivier Amédée-Manesme

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List of abbreviations

BNPPRE	BNP Paribas Real Estate
CBRE	Coldwell Banker Richard Ellis
EPRA	European Public Real Estate Association
PMA	Property Market Analysis
CAPM	Capital asset pricing model
INREV	European Association for Investors in Non-listed Real Estate Vehicles
IPD	Investment Property Databank
NCREIF	National Council of Real Estate Investment Fiduciaries
NOI	Net operating income
REITS	Real estate investments trusts
UCITs	Undertakings for Collective Investment in Transferable Securities
0.5% VaR	Value at Risk, 0.5% percentile
EU	European Union
US	United States
GAV	Gross Asset Value
NAV	Net Asset Value
LTV	Loan To Value
EIOPA	European Insurance and Occupational Pensions Authority (formerly CEIOPS)
CEIOPS	Committee of European Insurance and Occupational Pensions Supervisors
MRV	Market Rental Values
FCF	Free Cash Flows

GENERAL INTRODUCTION

Real estate is about land, and in that sense, real estate is a quarter of the earth's surface on which 6 billion humans live. Real estate is about built space - the space in which we live, work, walk and play - that design and define modern cities. Real estate represents a half of the value of all capital assets in the world[1] (source: The Economist, 2002[2]). No matter how real estate is defined, it cannot be ignored; real estate is fundamental.

Real estate is called real for a reason. Real estate is *real*, a topic of interest to everyone. It is built of concrete, steel and brick, and sometimes wood and straw. It has a strong influence on the lives of everyone, and it is of concern to everyone.

Real estate can be studied academically from many perspectives:

- Esthetic and functional (architecture)
- Physical structures (civil engineering)
- Ecologic (sustainable development)
- Urbanism (city construction and development)
- Legal (rights and duties associated with ownership)
- Sociologic (spacial and social phenomena)
- Economics (urban economics and civilization history)
- Finance (investment, portfolio management and risk valuation)

Even if not exhaustive, this list of disciplines is a good review of all topics spread out over the real estate arena. Real estate research appeals to multidisciplinary approaches, and encompasses numerous fields and researchers. This may explain why real estate faces difficulties when entering academic areas.

This thesis does not ignore these subjects, but is not intended to represent multidisciplinary research. Not only is contemporary real estate a crucial link to global capital markets and risk management, it is a critical resource in developed and developing economies. This resource requires expert analyses through academic and empirical research in all aspects of the topic. In this thesis, we connect, directly or indirectly, to: finance, economics, urban development, wealth accumulation, demography, environmental sustainability (linked to obsolescence) and public policies.

[1] In France, in 2011, and according to *Trésor Public*, real estate accounted for 60% of the country's wealth.
[2] The Economist 2002 was the only reliable reference we found. We faced difficulties finding recent valuations of wealth worldwide with emphasis on real estate. Research published by McKinsey exists, but we were unable to obtain it free.

To provide sufficient depth and rigor, we concentrate on one of the major disciplines in studying real estate: real estate finance. Real estate is a distinct asset class;[3] it is the major asset class for individual investors, the oldest asset class in history and is often presented as a fundamental asset class that should be included in every diversified portfolio (see Chaudhry et al., 1999; Hoesli et al., 2004; or Bekkers et al., 2009). In medieval times, land was the sole asset class considered important. Kings, Queens, Lords and Dukes measured wealth by the amount of land they held, and their names were often associated with the land they owned. Society changed in the last 400 years from one based on agriculture to one based on industry. Goods and services became new sources of wealth and power, so property wealth evolved with concentration on location. The question was decreasingly *how much* and increasingly *where*.

Real estate was an asset class long before all others. Stocks and bonds are modern investment innovations, created to support the needs of financial and industrial sectors, and more recently governments. It seems stocks and bonds became primary choices for investment instead of the classic choice of real estate.[4] It is often forgotten that real estate was society's primary asset class.

Before discussing analysis of real estate investments, portfolio and risk management, it is essential to gain understanding of land spaces and real estate. All of us have been involved with real estate our entire lives, but most of the time this involvement was unconscious. Real estate is comprised of two words, *real* and *estate*. Etymologically,[5] the word *real* refers to a thing in opposition to a person. This comes from the Latin word *res*, which means *matter* or *thing* (*réel* in Old French). The conceptual difference is between immovable property whose title transfers with the land, and movable property whose title can be removed from land. The word *estate* comes from the Old French word *estat* (*état* in modern French), which means position, condition, health or status. Etymology of the word makes a strong link between the tangibility of an asset and its condition. A more modern definition of real estate is the one from online dictionary Investorwords: real estate is "a piece of land, including the air above it and the ground below it, and any buildings or structures on it. Real estate can include business and/or residential properties, and are generally sold either by a realtor or directly by the individual who owns the property. In most situations […], real estate is a legal designation, and is subject to legislation." According to this definition, real estate is a piece of land, possibly with something on it.

[3] See Anson (2002).
[4] Now an alternative one.
[5] Source: http://www.etymonline.com/

Real estate covers a large number of assets with various property classifications, designs and features. Major real estate assets are land, retail properties, warehouses and industrial properties, office properties, hotels and residential properties.

- Land is the most basic type of real estate. Land can be used for farm, pastures or development. The size and location of a parcel of land are important in determining its value, developability and potential;
- Retail properties are properties used for a store, shopping center or service business. Some retail properties are downtown shops, owned individually. The primary difference with a shopping center is that each shop is interested in maximizing its own profit rather than maximizing profit for the entire retail environment. In contrast, a shopping center attempts to maximize the entire shopping experience through complementary tenancy, design, attractive features etc.;
- Warehouse and industrial properties are used for storage. They are generally simple physical structures, but they may have demanding specifications (floor slope, cooling, humidity, etc.). They are usually simple structures to allow easy reconfiguration;
- Office properties are buildings in which people work. Office properties can be located in central business districts or in suburban areas, and sometimes in the middle of nowhere. Central business districts are located in a city center near transportation networks and historic nodes. Suburban properties are located outside the city, and can generally be rented cheaply. They are usually assembled in parks of office building, with buildings sharing common amenities like a campus. In Europe, property location plays a larger role in determining not only value, but also category in comparison with states where design, age or amenities are key;
- Hotels are a niche. The category can provide full services such as restaurants and convention services or shopping, or can be limited. A hotel is an operating business with design and location components. Success depends on how well the property is operated;

- Residential properties are places for people to live. Types of housing include owner occupancy, tenancy, housing cooperatives, condominiums etc.

Varying property types serve different users, and require different leases, marketing, design and engineering. This is why real estate professionals are increasingly specializing in a particular property type. The industry evolved into numerous areas and jobs, and a non-exhaustive list of specialties includes: appraisal, brokerages, development, property management, real estate marketing, real estate investing, corporate real estate (managing the real estate held or used by a corporation) etc. Each field can be specialized into a type of property such as residential, office or retail.

Real estate became a major financial sector and investment asset over the past 30 years, referred to as commercial real estate. The *financiarization* of the sector in the 1980s in the United States and in the 1990s in Europe developed real estate quickly into a financial asset, creating real estate finance as a field itself.[6] Real estate finance is all about risk and opportunity. Property value is fundamentally an assessment of associated risk and opportunities. The primary risks faced in real estate investment are:

- Operating cost: operating cost is specific real estate risk. Utilities, property taxes, maintenance salaries, insurances etc. fluctuate during holding periods.
- Vacancy cost: a property may face periods of vacancy. Vacancies are one of the most important risks faced by real estate investors. A vacant property generates costs and becomes obsolete more quickly than a rented one.
- Obsolescence: a building does not keep its level of efficiency over time, requiring investment and maintenance.
- Leasing: a lease is a contract between a holder of property rights and a consumer or user holding at least some of those rights, covering a specific period. At the end of the contract, the space may become vacant. Leases may concede options to terminate the lease before the end of the contract (the option can be mutual or asymmetric).
- Liquidity: a building is less liquid than a government bond.

[6] France is an exception; real estate is often considered an external sector.

- Market risk: real estate suffers from two market risks: price risk (fluctuation of assets' prices) and user risk (consumption of space from tenants)

The primary opportunity in holding real estate is cash flow. While there are no cash flows opportunity associated with holding a 10-year AAA government bonds, there may be opportunities to generate cash flows from a building. Principally, these opportunities include:

- Operating costs: operating-cost synergies or expertise
- Terminal value: The value of the building at the end of a lease appreciates because of economic growth or inflation, allowing resell for profit. The property may be re-leased for a more favorable price to another tenant at the end of the contract.
- Rental growth: the lease provides a mechanism to increase total payments during the course of a lease.

A property therefore has greater advantages but also greater disadvantages than a 10-year AAA government bond, so an investor must adjust the discount rate - and risk valuation - accordingly.

To summarize this tentative definition, real estate is all things connected to land, its development, sale, purchase, construction, renting, and leasing; it includes everything that deals with properties.

Real estate investment involves the purchase, ownership, management and rental of real estate for profit. Real estate is an asset with limited liquidity relative to other investments. It is also capital intensive and highly cash-flow dependent. If an investor does not understand or manage these factors well, real estate is a risky investment. The primary cause of real estate investment failure occurs when the investment goes into negative cash flows for a period that is not sustainable, often forcing an investor to sell the property at a loss or risk insolvency. Investors add physical real estate to portfolios for many reasons. Some investors look for a hedge against inflation. Indeed, a portfolio of stocks and bonds suffers under inflation.[7] On the contrary, real estate and commodities are traditional hedges against inflation since their prices follow - or are even part of - inflation. Other investors add real estate to their portfolios for diversification. It has long been documented that a well-diversified portfolio lowers risk because it contains various asset classes that do not move in the

[7] This point raises various comments and it is not a subject of consensus among researchers. The literature review (part I.2) offers more details.

same direction at the same time. Real estate[8] offers investors the benefit of diversification, and this diversification produces regular income while other markets such as stocks decrease and pay small or no dividends. Lastly, investors seek real estate assets for decorrelation. Some argue real estate assets are decorrelated from traditional investment instruments. This assertion is controversial in real estate finance, and no consensus has emerged. There is no purpose to discussing it here, though real estate does not link perfectly to capital markets.[9]

In this thesis, the primary emphasis is financial economic aspects of real estate, and the most important aspect is to understand commercial property from an investment viewpoint. The central question when considering real estate investment is how and whether the asset will generate future cash flows. Academicians question real estate finance issues pertaining to the recurrence, magnitude, timing, security and nature of these cash flows. Cash flows are thus the most important drivers of real estate investment. Whatever the importance of cash flows and their nature, the fundamental question in real estate finance is risk, particularly with respect to cash-flow risk. This thesis concentrates on global risk: cash-flow risk, leasing risk, volatility, distribution and Value at Risk. More precisely, this thesis concentrates on one of the major - perhaps the most important - topic in finance: risk in real estate finance and how to estimate it reliably. Finance is all about risk, and real estate finance does not stray from this paradigm.

Over the years, finance has been controversial. Originally, finance determined opportunity costs and the value of investments, and allocated a flow of capital to and among underlying physical assets. Nevertheless, lack of regulations, increased debt burdens (overleveraging), financial innovations and complexities, and most importantly inaccurate pricing of risk steered the financial sector from its original purpose. This drift created controversies and criticisms within finance. Many economists consider the latest financial crisis to be the worst since the Great Depression of the 1930s, resulting in the collapse of large financial institutions, bank bailouts by national governments, and downturns in stock markets worldwide. In many areas,[10] the housing market suffered, resulting in evictions, foreclosures and prolonged unemployment. The crisis played a role in the failure of key businesses, declines in consumer wealth estimated in trillions of U.S. dollars, and a downturn in economic activity, leading to the 2008 to 2012 global recession and contributing to the European sovereign-debt crisis. Many causes of the financial crisis have been suggested, with varying weights assigned by

[8] We differentiate between physical real estate and listed real estate.

[9] See more discussions in the literature review.

[10] France is an exception.

experts, but the primary cause of the crisis was real estate. Bursting of the U.S. housing bubble, which peaked in February 2007, caused security values tied to U.S. real estate pricing to plummet, damaging financial institutions globally. Since then, finance has been the scapegoat.[11] Despite this irritation, we believe in the importance of finance in real estate. Physical structures and aesthetic characteristics studied by engineers and architects cannot exist without financial capital that commands and finances the resources to produce them. Therefore, the academic disciplines that study financial capital are of vital importance to real estate professionals.

Real estate finance is a mixture of corporate finance and investments, both relevant to analyzing commercial real estate assets. Before discussing the core of real estate finance and risk, it is necessary to address a few features specific to real estate. These topics were mentioned above but have not been defined. Real estate harbors nearly all problems encountered in the practice of finance, while at the same time they are traditionally part of more predictable assets. Methodologists argue there are two routes to understanding a phenomenon. The first consists of examining the general, the regular, and the ordinary, excluding the unusual. The second consists of examining problems and abnormal cases, excluding the ordinary. Both are useful in real estate. For those proceeding scientifically, real estate is an interesting asset since each possesses its own intricacies, subtleties and specifications:

- *Location* is the first exception. While a security is abstract - a simple balance-sheet entry - a property's location makes arbitrage arduous and comparisons difficult.[12]
- *Temporality.* A real estate asset is severely illiquid. Buying and selling are hardly unpredictable in real estate since they are grounded heavily to physical assets. Buying and selling spans months or years in comparison to securities that can be traded in seconds. A real estate investment is a long-term investment in comparison to traditional asset classes.
- *Investment size.* Real Estate assets are large, non-divisible assets. Investing in real estate is capital intensive, though capital can be gained through leverage. The minimum investment in real estate is larger than in other asset classes.
- *Obsolescence.* A building does not preserve efficiency. Thus, a building requires regular improvements and regular expenses to

[11] French translation: *bouc émissaire*

[12] A dictum in real estate: "the primary real estate drivers are location, location and location".

remain usable and current. A building is subject to legal restrictions in terms of external and internal changes. This point is essential since a building is a service provider: the higher the level of service, the higher the price paid for the service.

Real estate is a real asset on which specific financial tools must be built, developed and studied.

Even considering all of these points, academicians and researchers ask why a thesis in real estate contributes to the field. Although this question provokes and mocks, it is clear the research has to be motivated. Especially in France, real estate management appears to be a kind of loose canon of the academic research; it is part of economics and at the same time part of finance. Studying real estate leads to research in either economics or finance, so the traditional French differentiation between the two does not hold. Here, the objective is to motivate and justify real estate as a field and then to highlight the sub-areas of research in real estate finance tackle in this thesis. Finally, this section introduces and underlines areas that have not been subject to sufficient research and that can be examined more deeply.

Real estate is an academic discipline. Real estate and its sub-area commercial real estate are mature asset classes with identified investors and a relatively deep market. Certainly, real estate is an asset, but contrary to stocks and bonds, it requires physical management from owners or users. The tangibility of the asset attracts professional to work in this area. Second, following integration of physical commercial properties in institutional portfolios in the late 1980s in the United States, real estate became an academic discipline itself. Prior to that time, property research linked to geography and urban research. Now property research is a recognized, mature discipline with particular emphasis and interest on empirical and theoretical research. Third, real estate business and education are internationalizing. For a long time, real estate business was regional business, and so followed real estate education. Today, many real estate topics are treated from an international perspective, and academicians are increasingly interested in real estate markets abroad. The way an asset is surveyed, an investment opportunity approached and institutional investors diversify from home countries are of interest to academicians. Fourth, real estate assets account for 50% of worldwide wealth; that alone is a reason to conduct research on the topic. Fifth, particularly obvious in France and partially true elsewhere, real estate is not a purely academic discipline as are labor economics, industrial economics, derivatives pricing, quantitative finance and econometric theory. However, real estate research is developing, and that growth and a need for data help develop recognition for the field.

Extant real estate research focuses primarily on market modeling and forecasting, investment strategies, REITs,[13] appraisals, property development, urban economics, market theory equilibrium and indices. Like many other areas, the property sector has been affected by *financiarization*. Internationally, the real estate market has a profound effect on equity (pointed out later in this thesis); the subprime crisis of 2007 is one example. The market undergoes great fluctuations that spans months or years. Today, the real estate investing concept is undergoing change; from market-driven by owners and users, the market recently became an investor-driven asset, and *financiarization* in the industry is undergoing positive changes. Particularly, the emerging real estate derivatives market and the commercial real estate debt market attract new investors, and may lead to considerable changes in market structures. There is an area neglected: the underlying asset itself. Research focusing on the asset itself and its characteristics is scarce, due primarily to a lack of reliable data.

Many other research areas have not been the subject of sufficient research. This paragraph does not list all questions that can be addressed, but identifies three real estate domains that will require researches on in the future. First, there are opportunities for researchers interested in property risk management to provide descriptions, analyses and evaluations using contemporary approaches. The performance of standard methodologies in real estate is poor due to small databases and liquidity than in traditional investment sectors. This implicitly leads researchers to identify the best models to determine real estate risk. We present two new approaches for real estate VaR estimation. Second, published research on property derivatives, options and indices forget the portfolio insurance sector. Since producing pricing methodologies and indices in real estate represents a significant challenge, portfolio insurance is likely to be of interest to both academicians and professionals. This is an area absent of research in real estate finance. Third, real options are the subject of many papers, particularly in the context of property development. However, there is still a place for publication in this field for portfolio and property management. Many possibilities to use real options skills exist at the asset-management level, but there is still a large gap for development. Clearly, there is a strong scope of future research available.

[13] Real Estate Investment Trusts (REITs) are listed real estate vehicles.

One central issue we point out again is a lack of data in real estate finance research.[14] Some argue real estate is too complex and suffers heavily from a lack of relevant data to study the subject scientifically. Real estate research becomes more art than science. Real estate is complex, data are imperfect and decision-making is often a kind of art, but this state drives investment decision-making processes for half of the world's asset. At least for this half, studying real estate scientifically with available data is a sufficient reason.

This doctoral thesis focuses on commercial real estate finance risk, and is written in English. The basic problematic is how to better understand and better assess real estate risk. Another question can be: how to better value risk measurements? This thesis attempts to propose new models for risk valuation. It is thus a thesis based on models (by opposition to thesis based on empirical research and database). The proposed models seek to improve real estate risk understanding and valuation. This thesis is cumulative and consists of four papers:

1. Combining Monte Carlo Simulation and Options to Manage Risk of Real Estate Portfolio, chapter III;
2. Optimal Holding Periods of a Real Estate Portfolio according to the Leases, chapter IV;
3. Cornish-Fisher Expansion for Real Estate Value at Risk, chapter V;
4. Value at Risk: a Specific Real Estate Model, chapter VI;

In this introduction, we introduced and defined the real estate sector, and we motivated the topic of this thesis. Particularly, we partially answer the traditional question *why real estate*. The remainder of this dissertation is divided into three parts:

Part I - This part is an introduction to the remainder of the thesis. The first chapter describes briefly the real estate market, with an emphasis on the European real estate market.[15] The market description is introductory and not intended to be exhaustive. The second chapter presents an account of what has been published in real estate finance by scholars and researchers. The objective of the chapter is not to detail all publications, but convey established knowledge and ideas of interest to this research.

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[14] Much real estate data come from private operators. As a result, data are infrequent, elusive, fragmented and expensive. In particular, data are difficult to standardize, very localized, confidential and often unavailable.

[15] Data were more easily available in Europe.

Part II - This part focuses on real estate portfolio and risk management, with emphasis on both options embedded in real estate leasing structures and market risk. We determine how lease structures influence portfolio valuation, portfolio risk and portfolio management. The first section introduces real estate portfolio management and explores issues arising from traditional methodologies. Then we present a model that integrates break clauses offered to tenants. This is done with simultaneously use of Monte Carlo simulations and options theory. The third section turns to holding-period strategy in a real estate portfolio when options embedded in leases are considered. This work relies on the first article presented in chapter three, and is a continuation of a previous article by Baroni et al. (2007b). The problems, objectives, methodologies and scientific contributions of the two chapters are presented.

Part III - This part deals with risk and, more specifically, Value at Risk measurement. Value at Risk is a risk measurement whose computation is now mandatory for regulatory purpose (Bale II, Bale III, Solvency II and NAIC).[16] In real estate, no model has been proposed to compute Value at Risk reliably. Nevertheless, real estate includes specification data that are considered better when evaluating investment Value at Risk. This part proposes a chapter that concentrates on Value at Risk computations using Cornish-Fisher and a rearrangement procedure. The traditional assumption of normality can be overtaken. A second chapter (chapter VI) presents a model of Value at Risk assessment that considers primary real estate specifications. Scientific contributions and methodologies are explained.

Both papers presented in these two chapters were lauded by the European Real Estate Society conference during the PhD session.

[16] The National Association of Insurance Commissioners (NAIC) is the U.S. standard-setting and regulatory support organization created and governed by the chief insurance regulators for all 50 states (equivalent to the European Insurance and Occupational Pensions Authority—the EIOPA—for Europe, which promotes Solvency II regulations.

PART I – REAL ESTATE MARKET AND LITERATURE

Chapter I. A brief review of real estate market

A review of the real estate market is challenging, but useful for researchers to understand the market in which they evolve. This is particularly true because the real estate market is neither observable nor transparent. The countries included in this market presentation are fixed technically by the availability of information. The following information comes primarily from BNP Paribas Real Estate, CBRE, INREV, EPRA, PMA, IPD, and other sources. A book by Suarez (2008), reviewing European real estate markets, was great help.

This market presentation covers two topics: a description of the real estate market and the financial activity linked to real estate investment.

Real estate is one of the primary activity sectors worldwide. Property markets have many impacts on various players, from individuals to investors. A large part of an individual's wealth in the world is invested in real estate assets. In France, according to the state agency,[17] real estate represents 61% of wealth in the country. Worldwide, real estate assets represent 54% of wealth and financial assets represent 46%. These numbers appeared in The Economist newspapers. Developed economy assets at the end of 2002 were:

[17] *Direction du Trésor*

- Residential property: $48 trillion
- Commercial property: $14 trillion
- Equities: $20 trillion
- Government bonds: $20 trillion
- Corporate bonds: $13 trillion

- **Total: $115 trillion**[18]

Graph I-1 - Market sector value
Source: The Economist, 2002

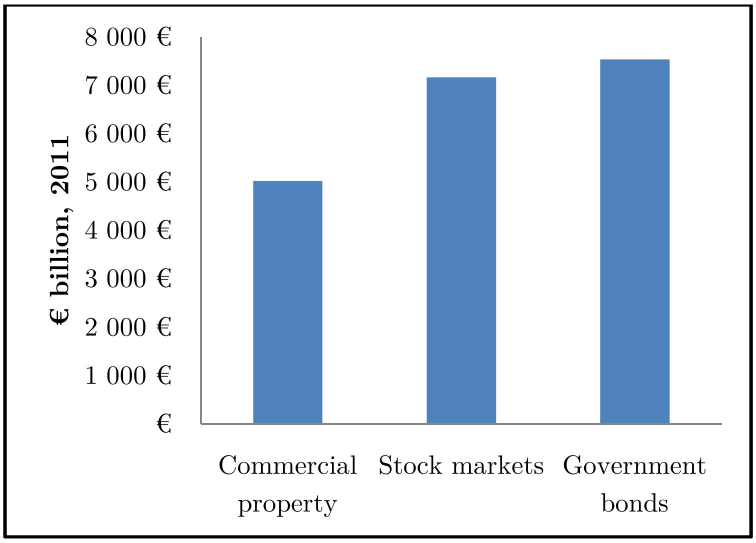
The real estate sector influences activities in financial institutions. A simple look at what happened during the second half of 2007 in the financial market when U.S. real estate markets started to collapse demonstrates a strong relationship between real estate and other financial sectors, and more generally other economic sectors. The financial crisis was triggered primarily by subprime mortgage loans. A large portion of financial institutions' balance sheets is made up of mortgage and property-related loans. These loans had experienced huge growth worldwide until 2007, and some European countries such as Spain experienced this growth until 2008. Today, companies are still clearing balance sheets of loans and mortgages that collapsed since then.

The real estate industry also contributes to activities in financial markets in new ways, with the real estate derivatives market a notable example. Derivatives on property assets are a recent innovation and may play a role in the management of real estate investment risk in the future. In addition, many financial products deriving from real estate are booming. For example, commercial real estate debt (as opposition to corporate debt) is an old product, but with recent interest coming from institutional investors. The volume invested by institutions increased considerably in recent years, particularly in light of Solvency II regulations for insurers.[19] However, rapid growth of new securities, vehicles and products, and their complexities, led many investors to take positions they did not fully understand, with risk management systems that do not cover all scenarios. New risk management systems and measurements, especially built on the real estate sector, are needed. This thesis attempts to fill this gap.

Commercial real estate plays a role in business, accounting for a large portion of companies' investments and expenses. Above all, it is one of the primary

[18] Assets not counted include bank deposits, insurance "reserve" assets, natural resources and human assets.
[19] Contrarily, due to capital requirements, banks are reducing exposure to real estate.

investments of professional and institutional investors in real estate. For illustration, EPRA/INREV[20] computed the size of European real estate, displayed in Graph I-2.



Graph I-2 – European commercial property market size
Source: EPRA/INREEV research, 2012

Commercial property other than residential (shops and retail, offices, industrial) plays a vital role in Europe's business, industry and social life. Its market value in 2011 was approximately € 5 trillion, comparable to the value of plant and machinery in Europe's businesses and close to the size of European stock and government bond markets. According to EPRA, the value of housing (residential) at € 22.5 trillion far exceeds other property or investment sectors.

During the past 15 years (20 years in the U.K.), a large dissemination of information concerning real estate transactions and information boosted the confidence and transparency of the property sector as a whole throughout Europe. This information takes the form of investment and rental reports from brokers, asset managers and business associations. This dissemination was further helped by the creation of property benchmarks such as INREV, IPD and PMA.[21] One consequence[22] of the growth of transparency and available information was the number of investors

[20] EPRA is the European Public Real Estate Association, and INREV is the European Association for Investors in Non-listed Real Estate Vehicles.
[21] IPD is the Investment Property Database, and PMA is the Property Market Analysis.
[22] Another consequence of real estate transparency improvement is growth in required management of properties, projects and assets. Property investors must hire real estate specialists to accomplish tasks required for preservation of an asset.

who crossed frontiers. With improvements in market transparency, foreign investments in all countries emerged.

Real estate is viewed historically as a local phenomenon. Over many years, builders and developers relied on themselves to find the best land and property. This belief was based on local knowledge. Combined with the fact that real estate assets are unmovable physically, the real estate industry was one of the last to globalize.[23] Until the 1990s and despite occasional deals, the business itself remained largely local. In the last decade, globalization increasingly involved internationalization of services sectors, among which lays the real estate industry. Builders, brokerage firms, consulting and services firms, real estate finance firms and investors extended their areas of operation beyond local markets to a worldwide base. This point is fundamental at the time of the writing of this thesis. One obvious signal of this growing globalization is growth of cross-country investment. In France in 2011,[24] 42% of investments came from foreign investors. Another sign of globalization is the growing number of companies in the real estate sector crossing the frontier to conduct international business operations, such as various American or Qataris funds, and other pension funds. Although increasing internationalization is standardizing practices - and to some extent returns - low correlation is observable in investment returns in various countries. This is illustrated below by the correlation matrix of European countries using PMA Database.

[23] Some economists still argue the real estate industry is local.

[24] Source: BNP Paribas Real Estate.

	Brussels	Paris: CBD	Berlin	Frankfurt: City	Milan	London: Central	London: West End and Midtown
Brussels	100%	60%	-38%	-22%	81%	58%	58%
Paris: CBD		100%	7%	19%	41%	87%	85%
Berlin			100%	88%	-40%	-9%	-17%
Frankfurt: City				100%	-8%	0%	-10%
Milan					100%	40%	41%
London: Central						100%	99%
London: West End and Midtown							100%

Source: PMA Database, 2000-2010

a. 1989 to 2010

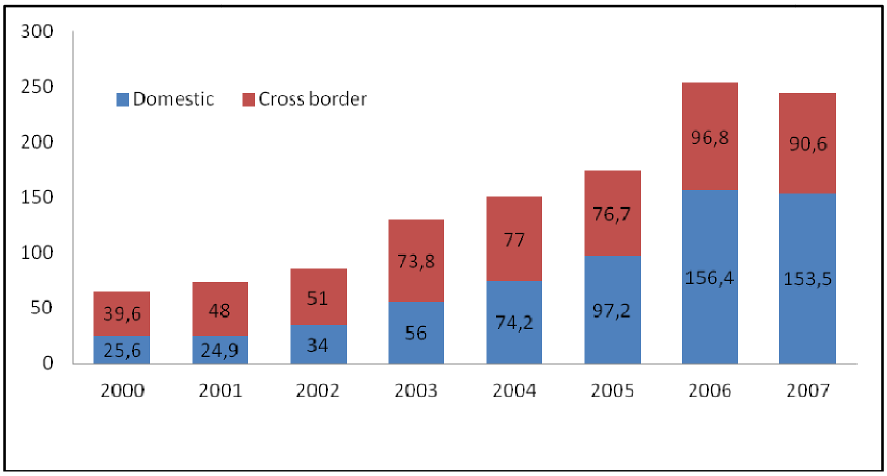
	Brussels	Paris: CBD	Berlin	Frankfurt: City	Milan	London: Central	London: West End and Midtown
Brussels	100%	70%	-23%	17%	93%	70%	76%
Paris: CBD		100%	-11%	40%	78%	76%	75%
Berlin			100%	59%	-30%	-58%	-58%
Frankfurt: City				100%	24%	-4%	-6%
Milan					100%	71%	75%
London: Central						100%	99%
London: West End and Midtown							100%

Source: PMA Database, 1989-2010

b. 2000 to 2010

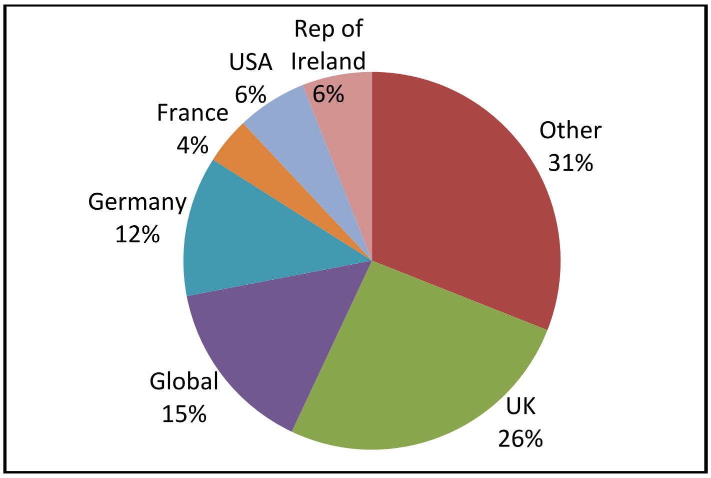
Table I-1 - Correlation matrix between some real estate markets over two periods

In addition to globalization, regulation of numerous activities is converging across countries, particularly in Europe, and the real estate investment trust (REIT) is an example of this convergence. REITs began in the 1960s in the United States and are now present in nearly all developed countries. They were introduced in France in 2003 and in the U.K., Germany and Italy in 2007. Another example is growth of real estate companies and funds, particularly growth of assets managed by fund or portfolio managers. They require geographic diversification to meet growth and diversification goals. Graph I-3 and Graph I-4 illustrate this growth for European countries. Thus, many factors drive growing internationalization.



Graph I-3 - Direct real estate investment volume, M€, Europe, 2007

Source: Jones Lang Lasalle (2007) & Suarez (2008)



Graph I-4 - Source of capital invested in European real estate, 2007, percentage of the total

Source: Jones Lang Lasalle (2007) & Suarez (2008)

Globalization has particularly appeared in Europe where many companies already operate in more than one country, helped greatly by the common money: the euro. In addition, practices and procedures are becoming increasingly similar following European regulation (UCITs for instance).

Institutions on other continents also contributed to internationalization in Europe. American real estate companies and gulf countries played a role in the development of commercial real estate businesses, strategy and development in Europe.

I. The real estate market

Real estate is classified generally into three main areas:

- Owner-occupied houses;
- Commercial real estate;
- Special real estate assets[25]

We focus on commercial real estate. This thesis is driven primarily by institutional investors, and commercial real estate is nearly the only sector of investment in real estate for professional investors due to size, competition and returns. Commercial real estate includes many types of assets. The main properties by value and transactions include offices, retail, industrial and logistics, but this category also includes hotels, apartment blocks, parking lots, retirement homes, student homes, etc. Due to availability of data,[26] we define three sectors: offices, shopping centers and industrial/logistics.

- Offices are the most typical investment assets in commercial real estate. It is the largest sector and the most liquid. They are easier to manage, usually requiring relatively little effort to rent and attract quality tenants. Graph I-5 shows offices as the predominant sector in commercial real estate.
- Shopping centers are retail properties that are planned, built and managed as a single entity, comprising units and communal areas with a minimum gross rentable area of 5,000 sqm (International Council of Shopping Centers reports, 2005). Shopping centers can be a general shopping center or a specialized one. They can be classified by size or other criteria such as outlets, themes etc.
- Industrial/logistic properties are built for the management of resources, between a point of origin and a point of destination for the logistic sector, and for transformation and management of the resource itself for the industrial sector. The industrial and logistic markets are influenced by economic activity in each

[25] Special real estate includes churches, public buildings and facilities. It is an emerging market, and we did not find conclusive data on this point. Particularly in France, public real estate holdings or management - among which include universities - is incipient and will certainly rise in coming years.

[26] Commercial real estate information is normally presented by city instead of country.

geographic area, and by adjacent areas, particularly with regard to production and international commerce.

When considering commercial real estate, one should first establish the size of stock, then the rental market and its principal economic variables (take-up,[27] rent and occupancy), and finally the investment market (transactions, prices and yield).[28] These data are not presented here for simplicity of presentation. However, interested readers can find this information easily and freely on brokers or asset managers' websites (CBRE, BNP Paribas Real Estate, ING and Jones Lang Lasalle etc.) for all major cities worldwide.

Existing property stock can be measured by size (square meters/feet) or investment value. After deciding on size or investment value, either total stock constructed and extant or investible stock can be considered. In 2012, Europe was the continent with the second largest real estate stock[29] after North America, and as expected, the majority of this stock is located in Western Europe. The largest markets in terms of country are the United States and Japan. As shown in Table I-2, since the mid-1990s, investment volumes and stocks increased in European real estate markets. Among other reasons, this was due to the industry's recurring income (rent) and, as mentioned above, low correlation of direct real estate with equity and fixed-income investments. Improved market accessibility also encouraged many investors to invest in real estate assets. All of these factors led institutional investors to increase the weights of real estate assets in investment portfolios.

[27] Take-up represents the amount of sqm let over a past period (quarter, year etc.).
[28] Yield is immediate profitability from a property, measured as the ratio of net income to property price; net income is rent minus expenses incurred.
[29] We did not find consistent data concerning Chinese and Indian markets.

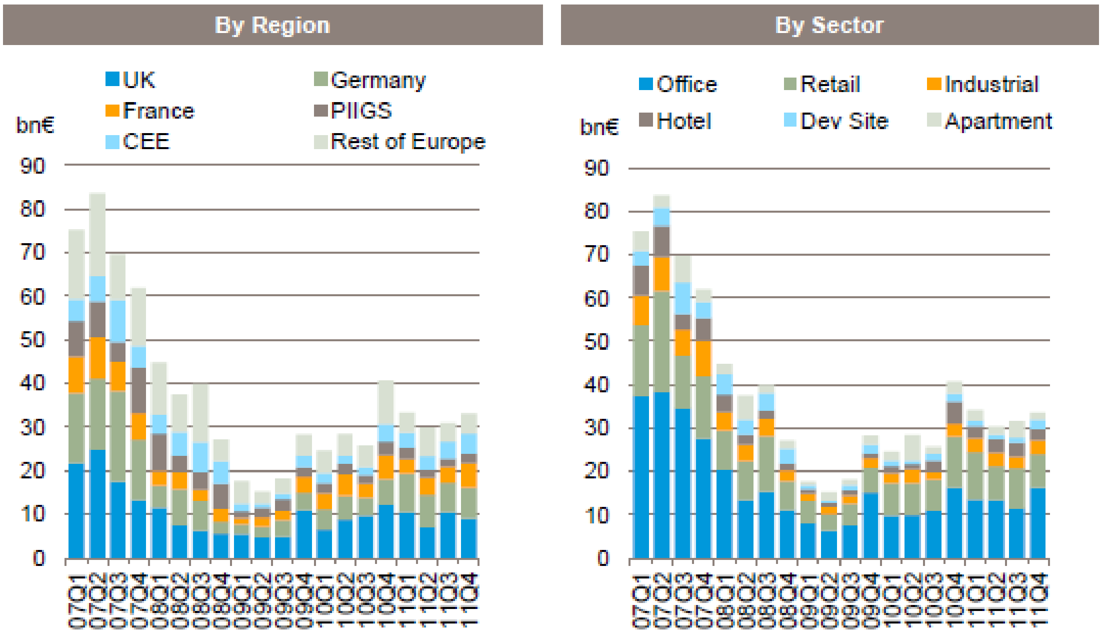
Country	1998	2006	2011
Australia	96 271	153 895	182 509
Austria	-	17 387	22 786
Belgium	-	29 765	47 610
Canada	-	98 619	139 822
Czech Republic	-	12 411	11 152
Denmark	-	26 890	34 425
Finland (KTI)	25 736	27 550	42 900
France	112 499	181 830	236 332
Germany	317 948	278 139	270 341
Ireland	2 888	7 177	2 522
Italy	-	62 845	79 475
Japan	-	207 244	595 735
Korea	-	13 776	38 149
Netherlands	55 998	81 645	117 220
Norway	-	27 677	40 237
Poland	-	11 357	16 224
Portugal	-	13 028	15 073
South Africa	14 921	18 344	30 875
Spain	-	31 877	40 984
Sweden	55 241	80 352	107 345
Switzerland	-	97 604	140 231
UK	277 533	486 182	284 085
US	-	1 193 535	1 516 190

Table I-2 - Estimated size of total market (M€)

Source: IPD, 2012

Stated earlier, in dealing with commercial real estate, there are two markets: rental markets (take-up) and investment markets, and investors usually consider both. First, they consider take-up - the square meters let during the past year or quarter - and they compare it to available vacant stock (or soon vacant). Second, they consider the investment in the sector and compare it to other transactions. It is essential for an investor to buy property that can be let, and to buy it at a price comparable to the market.[30]

[30] In real estate, one or two transactions can represent 20% to 30% of the market, and are not necessarily representative of the market as a whole. This is why comparables are difficult to find and analyze.



Graph I-5 - European Investment Volume
Source: Real Capital Analytics, February 2011

Three countries dominate European real estate: Germany, France and the U.K. The largest European market is the U.K. (Table I-2 and Graph I-5). Graph I-5 shows some other related facts. Investment volumes have reduced greatly since 2007, with opportunistic investments nearly inexistent (Dev Site on the Graph I-5). In terms of investment, the big three European markets dominate European investments. In term of sectors, office[31] and retail lead the market with more than 65% of transactions since 2007. Financial turmoil has had a strong impact on the property market; the volume invested reduced to half since 2007.

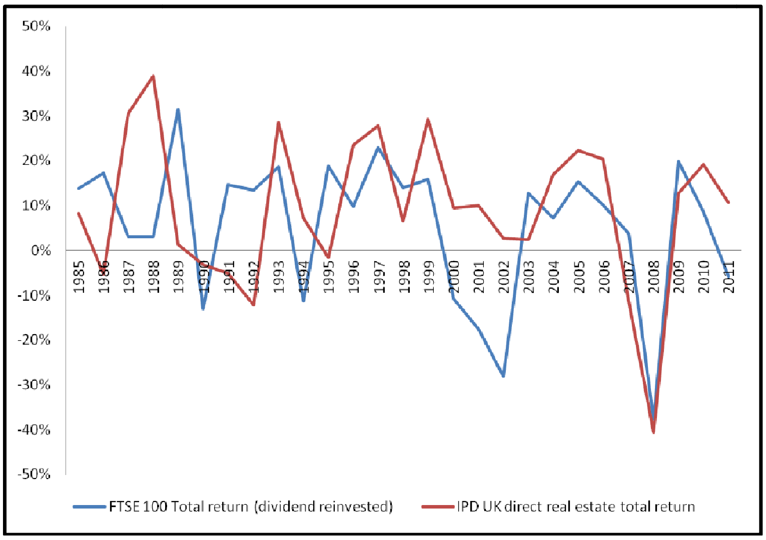
II. The financial activity: direct and indirect investment

A. Direct Real Estate

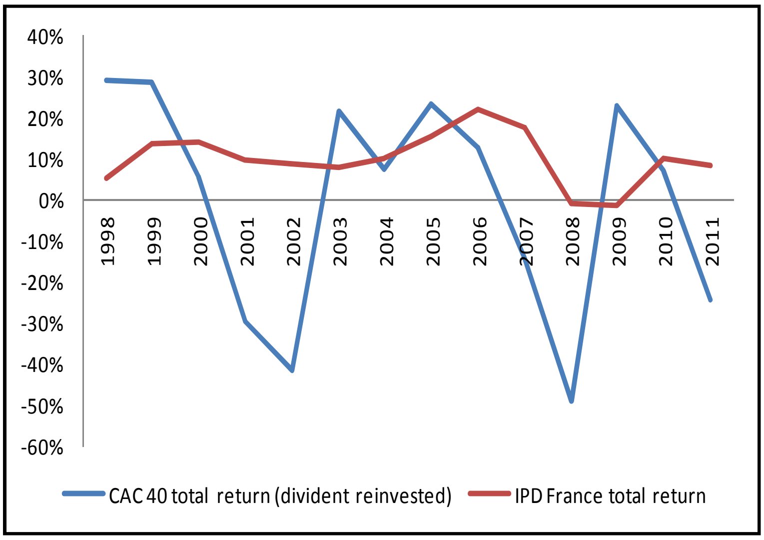
Direct real estate involves the purchase, ownership, management, rental and/or sale of real estate for profit. Improvements, redevelopments and repositionings are part of real estate investing. Direct real estate is a specific investment typology that

[31] The office sector is more liquid and easier in which to invest.

involves limited liquidity relative to other investments. It is capital intensive and highly cash-flow dependent. If all specifications are not understood and managed well, direct real estate becomes a risky investment. The most distinct characteristic of direct real estate investment is low liquidity; property is not a readily marketable asset like equities or bonds. Direct real estate returns are much less convertible than equity returns: property is a less volatile asset. Some real estate academicians (Holmstrom and Tirolle, 2001 or Danielsen et al., 2010) argue that the high volatility of equity investment is the price paid for greater liquidity. This assertion is true for all direct real estate markets except in the U.K. where real estate assets are as volatile as equity markets, or nearly so. However, volatility is paid for by a higher return, and therefore risk-return profiles of the direct real estate market in the U.K. is better than those in equity markets, if computed without liquidity premiums.



Graph I-6 - FTSE return versus IPD U.K. total return



Graph I-7 - CAC return versus IPD France total return

One of the first drivers of direct real estate investors is a search for higher returns. Between 1998 and 2011, the average real estate return was 8% in the U.K. and 10% in France, while the average annual return was 1% for the FSTE and -0.1% for the CAC. In terms of volatility, it is clear that real estate is less volatile in France than the stock market (5% versus 25%), as illustrated in Table I-3. Direct real estate exhibits higher returns, lower volatility and low correlation, which makes it one of the best portfolio diversifiers. It is important to understand that we do not consider premiums for liquidity issues, though many researchers argue that institutional investors account for this liquidity issue by underweighting real estate assets in portfolios (Garlappi et al., 2007; Bond and Slezak, 2010).

		FTSE 100	IPD UK Total return
Return	1985-2006	7,4%	11,8%
	1985-2011	5,6%	9,3%
Volatility	1985-2006	14,7%	14,1%
	1985-2011	16,2%	16,8%
Correlations	1985-2006	8,7%	
	1985-2011	38,6%	

Table I-3 - Basic statistics of FTSE and IPD U.K.

		FTSE 100	IPD France Total return
Return	1998-2006	6,3%	11,8%
	1998-2011	-0,1%	10,0%
Volatility	1998-2006	25,5%	5,0%
	1998-2011	26,8%	6,5%
Correlation	1998-2006	18,1%	
	1998-2011	21,9%	

Table I-4 - Basic statistics of CAC and IPD France

Direct real estate investment is also characterized by high transaction costs, both for the buyer and seller, and high administrative and management costs. In addition to disparate taxes, change in ownership normally implies legal and brokerage expenses. Also, real estate assets are heterogeneous: one asset is not interchangeable with another. Traditionally, direct real estate is driven by three factors: location, location and location. However, the industry is becoming more complex, and location is no longer the only factor in property returns. For example, the benefits of leverage and tax deductions, given complex-structured products, allow higher returns.

Direct real estate is a distinct asset requiring active management and competencies (technical, legal and financial) from investors. These points are counterbalanced with better returns and lower volatility, except in the U.K. according to available data.

B. Indirect Investment in Real Estate

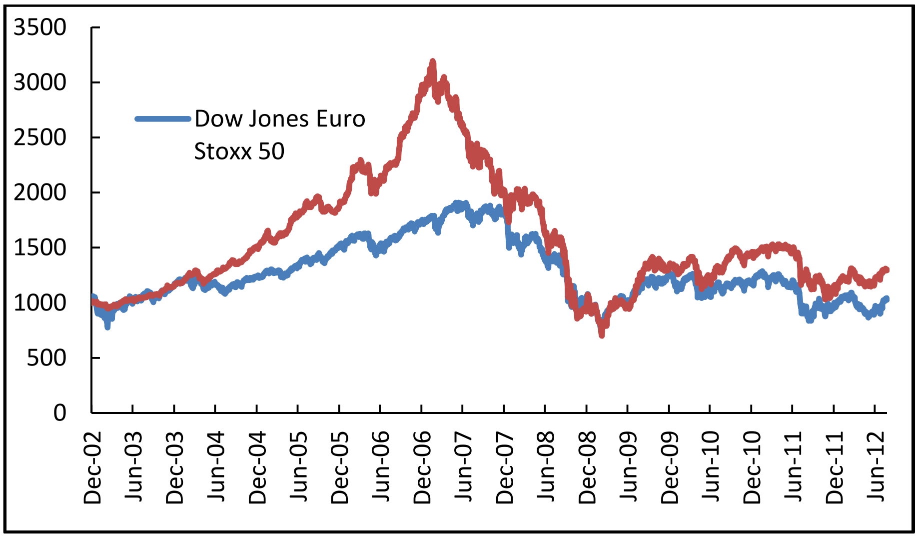
There are two primary vehicles for indirect investment in real estate: real estate investment funds and listed real estate companies, commonly called REITs. Indirect investments offer many advantages over direct investments. Indirect investors benefit from scale effects (indirect investments allow for smaller investment), diversification (an investment in an indirect vehicle generally leads to a share of many properties) and professional management that help investors choose suitable properties.

Listed real estate offers advantages in comparison to direct real estate, but these benefits apply to other indirect real estate vehicles, among them funds.[32] The most important benefit is tax advantages for an investor; in most countries, listed real

[32] except SCPI and OPCI in France

estate is a flow-through vehicle for tax purposes,[33] and legislation usually requires distribution of between 75% to 90% of profit and a minimum investment in property greater than 60%. In addition, listed real estate is a flexible investment; changing sectors or regional weights span months in direct or funds investment, and may take only a few days in a listed portfolio. There are also disadvantages to indirect real estate. For example, stock market movements affect listed real estate, and they generally exhibit greater volatility. Some argue confidentiality of fund management and opacity of decisions are other issues.

Real estate stock performance has two components, one related to the stock markets and the other to evolution of property markets. Graph I-8 shows the performance of European real estate stocks juxtaposed to stock markets, represented by the Eurostoxx 50. The returns and volatilities of real estate are computed in Table I-5. Listed real estate is a very risky (volatile) asset in comparison to direct real estate.



Graph I-8 - Listed European real estate (Eurozone) versus Eurostoxx 50

Source: IEIF

[33] Avoiding double taxation make REITs (listed real estate) tax-efficient investments.

31/07/2012	**Return**
	5 years
REIT Europe	-5,44%
REIT Continental Europe	-5,94%
Dow Jones Euro Stoxx 50	-8,51%

31/07/2012	**Volatility**	
	1 year	**3 years**
REIT Europe	26,06%	16,69%
REIT Continental Europe	106,42%	16,38%
Dow Jones Euro Stoxx 50	30,78%	18,75%

Table I-5 - Comparison of volatility and returns between REITs and Eurostoxx 50
Source: IEIF

In this thesis, we do not compare direct and indirect real estate performance. This subject is a thesis in itself, and has been the focus of much research. We focus on direct real estate investments even if results can be applied and generalized to listed real estate.

Chapter II. Review of the literature and recent research developments

A researcher cannot perform significant research without first understanding the literature in the field

Boote and Beile (2005)

A literature review is a review of what has been published on a topic by academicians, researchers and sometimes practitioners. This literature review takes a place in the introduction of the thesis. The intention in writing this literature review is more to convey knowledge and ideas established in real estate finance than to conduct an exhaustive literature review. The real estate finance literature is sparse and relatively recent.[1] The real estate finance industry emerged in the 1980s, and academicians in the field started about the same time; in Europe, the real estate finance industry emerged later in the 1990s.

Before starting this literature review, it is fundamental to cite four books on which the fundaments of this thesis rely. These books can be seen as the bedside books of this dissertation. Without these books, the thesis would have been more difficult to write. The books are those of Portrait and Poncet (2011), Vernimmen et al. (2013), Poncet and Aftalion (2003) and Prigent et al. (2007).[2]

The purpose of the literature review is not to provide a descriptive list of available research or create summaries and presentations of the concepts and work on topics we explore here. We organize this review around topics related directly to the research question. We synthesize primary results and highlight the most important research. We nearly avoid controversial areas in the literature because our research

[1] In comparison to other financial sectors, in which the literature started in the 1950s.

[2] See also Bertrand and Prigent (2012), in French.

area is not controversial with respect to results.[3] Whenever possible, we formulate questions that require further attention in the future for researchers and practitioners.

This literature review attempts to be broad, describing numerous fields of real estate finance and economics. We particularly concentrate on the topic we address now and conclude on later in the thesis. For this reason, some repetition occurs between this literature review and the literature review concerning the other papers. We minimize the number of repeated studies and academic works, but some researchers and papers are fundamental and are mentioned two or three times in the thesis. In advance, we ask the reader to forgive these repetitions.

The review is built around four parts. First, we present literature related to real estate portfolio management. We introduce numerous studies but concentrate on real estate as a portfolio diversifier, the traditional debate on sector versus regional diversification in the context of a real estate portfolio, inflation hedging properties of real estate and optimal debt in a real estate fund. We complete this presentation with an introduction to real estate derivatives research. Second, we introduce lease and lease structures, and present research conducted in this field. We particularly highlight terms and various types of leases. Third, we review distributions in real estate, particularly of return distributions. We present all primary contributions from real estate academicians in the field. Unfortunately, these contributions come primarily from the United States and U.K. markets, likely based on data availability. Finally, we concentrate on Value at Risk. We present the sparse research on the topic conducted in real estate fields, and those in general finance fields. All missing references or mistakes - if any - are those of the authors.

I. Real estate portfolio allocation and portfolio management

Most extant studies agree on the importance of strategic asset allocation as a fundamental parameter of investment returns. Brinson et al. (1986) reveal that 93% of performance differences can be explained with assets allocation, implying strategic asset allocation is far more important than market timing and security selection. Portfolio diversification and allocation have long been of interest to real estate academicians. The most important authors in this area are, among others: Byrne and Lee (1996, 1997, 1998, 2000, 2011) and Hoesli et al. (2000, 2004, 2012). The literature

[3] For a controversial area, see research concerning copulas.

in this field is large, and presentation is restricted to only three of the primary trends. These trends are the role of real estate as a portfolio diversifier, the tradeoff between region or sector diversification and the debate involving number of leases versus number of assets in portfolio diversification. We discuss briefly inflation hedging characteristics of real estate, and optimal debt level in a fund. We conclude this review with a note on real estate derivatives.

A. Real estate as a portfolio diversifier

Traditionally, real estate assets are regarded as safe investments, with inflation-hedging properties that offer diversification potential and high absolute returns. Nevertheless, there exists no consensus on its role in an investment context. Many papers focus on the benefits of including real estate assets in mixed-asset portfolios.[4] Real estate has long been proven an effective portfolio diversifier at both domestic and international levels. Optimal allocation to real estate is generally found - according to authors, countries and databases - to stand from 15% to 25%. This level remains stable when the standard deviation varies. Optimal allocation of an institutional portfolio among various asset classes is of primary importance. In fact, it has a critical impact on portfolio risk and performance.

The positive role of real estate in a mixed-asset portfolio supposes real estate returns have low correlations with stocks and bonds. Using co-integration techniques, Chaudhry et al. (1999) demonstrate stocks correlate negatively with real estate in the long-term. They also show that stocks have a weaker impact on real estate assets than on bonds and T-bills. Concentrating on risk reductions brought on by real estate, Hoesli et al. (2004) analyze the impact of including direct real estate in portfolios of financial assets for seven countries at domestic and international levels.[5] They address data-quality issues: appraisal-based indices and mean returns. Findings suggest adding real estate at the domestic level leads to a 5% to 10% risk reduction, and at the international level, the reduction is 10% to 20%. Surprisingly, the authors found a quasi-constant, optimal real estate allocation: 15% to 25%. They found nearly the same results whether currencies are hedged. Recently Bekkers et al. (2009) explore which asset classes add value to a traditional portfolio of stocks, bonds and cash, and determine optimal weights. Results recommend adding real estate, commodities and

[4] See also Hudson et al. (2003)

[5] They particularly consider currency hedging issues.

high yield bonds to the traditional asset mix of corporate and government bonds and stocks. MacKinnon and Zaman (2009) examine the diversification properties of real estate according investor time horizons, and consider return predictability. Results demonstrate the risk of real estate returns is much less for long-term investors than for short-term investors because of mean-reversion properties of real estate returns. However, from their findings, long-term risk characteristics of direct real estate are close to those of stocks. They conclude that diversification benefits of real estate come from correlations that decrease on a long horizon. In addition to these papers using correlations and traditional portfolio allocation, Brounen et al. (2010) consider liabilities of investors in portfolio allocation. They demonstrate direct real estate assets should be included in a portfolio when investors consider liabilities for diversification potential and not for inflation-hedging properties. Brounen et al. (op. cit.) determine an optimal portfolio allocation for real estate ranges from 16% to 35% according to the expected return utility and depending on the level of risk tolerance. Rehring (2012) compares portfolio choice results for alternative asset allocation approaches, considering marketing period risk, returns predictability and transaction costs.[6] The paper introduces an asset allocation approach developed by Campbell and Viceira (2002) to estimate the term structure of risk. The author highlights differences among short, medium and long-term investments, concluding by showing that the usual mean-variance analysis can be misleading because it ignores returns predictability, transaction costs and marketing period risk; it overweighs real estate in portfolio allocation.

Some researchers concentrate on international diversification. Strong evidence suggests international diversification is useful for real estate (as shown in the market review, decorrelation is more important). One of the first papers in this field was Eicholtz (1996). Eicholtz showed how international diversification works better for real estate (indirect) than for stocks and bonds. In the same trend, Gordon et al. (1998) found that the efficient frontier containing international listed real estate dominates the frontier with international stocks and indirect real estate in a U.S. context. Case et al. (2000) show how international direct real estate diversification is useful to U.S. investors. Professional real estate companies that aimed to justify suggested allocations largely published numerous recent studies on the topic.

Diversification using listed real estate (REITs) as a direct real estate proxy is a traditional question in portfolio allocation. Are REITs real estate or stocks? Many

[6] Transaction costs are important for the weight assigned to real estate in the short to medium term.

studies examine the question of whether securitized real estate returns reflect direct real estate returns of general stocks market returns. Direct real estate has shown to provide diversification benefits. However, it exhibits several disadvantages such as low liquidity, high transaction costs and lumpiness, particularly when investment exists across a fund. Securitized and direct real estate might be driven by common real estate factors over long horizon; therefore listed real estate is expected to provide the same diversification benefits as the direct one. The question of whether real estate securities behave as real estate or equities is old and fundamental for a large number of investors. The answer is not conclusive in extant literature. Among others, the primary contributions to the field include Goetzmann and Ibbotson (1990), Ross and Zisler (1991), Myer and Webb (1994), Mei and Lee (1994), Mueller and Mueller (2003), Brounen and Eicholtz (2001), Schätz and Sebastian (2009), Yunus (2009), Pavlov and Wachter (2011) and more recently Hoesli and Oikarinen (2012).

Even with all this research, institutional investors allocate lower weights to real estate than those suggested by literature. Clayton (2007) estimates allocation to real estate by institutional investors is 7.3% in the United States and 8.5% in the United Kingdom. Chun et al. (2004)[7] underlines how real estate researchers can question differences between suggested allocations and low actual allocations to real estate in portfolios of institutional investors. According to Chun et al. (op. cit.), it seems institutional investors are not solely in search of maximum returns; they might consider risk relative to liabilities, and consider risk-adjusted measurements. Numerous extant studies are unable to solve this contradiction, and demonstrate inefficiency in understanding why. Among others, Chun et al. (2000), Bond et al. (2008) and Geltner et al. (2006) highlight the difference between suggested and real allocation, but do not explain it. Some studies try to explain this observation. Kallberg et al. (1996) and Chun et al. (2000) proposed explanations earlier. Chun et al. (op. cit.) supposed that institutional investors do not use asset optimization but asset liability optimization to determine weights in a portfolio. Their work focuses on indirect listed real estate only (REITs). Kallberg, et al. (1996) argue institutional investors consider real estate specifications and difficulties when determining portfolio allocation, such as indivisible assets, illiquidity,[8] no possibility of short-selling and local specificities. More recent research confirms the intuition of Chun et al. (2000). Brounen et al. (2010)[9] show that the weight that should be allocated to real estate is in line with actual institutional

[7] See also Chun et al. (2008)

[8] See Cao and Teiletche (2007) for ways to deal with estimation problems of illiquid assets.

[9] See also Brounen et al. (2009).

weights if an asset-liability management framework is used rather than traditional models. Second, the same group of authors, Chun et al. (2004), suggests traditional, modern portfolio theory is difficult to apply in real estate because returns are dependent and are not distributed identically.

B. Sector versus region

A large number of papers, written largely in the 1990s, took an interest in diversification in real estate, more precisely on sector versus regional diversification issues, from a number of perspectives. Today, when real estate investors want to diversify portfolios, it is still done through a process of naïve sector versus region diversification. From this comes the problem of whether investors should confine investments to one region and seek diversification by real estate sector within the region, or diversify across regions while remaining within a real estate sector.[10]

One of the first papers focusing on this area was Eichholtz et al. (1995) in which the authors examine the benefits of sector versus regional diversification in the U.S. and U.K. They use various methods including correlation analysis, principal components and mean-variance analysis. For the U.S., they find that retail investment should be diversified across regions and that industrial and office investment should be diversified across real estate types. For the U.K., the opposite was true. Retail should be diversified across sector, and office and industrial should be diversified across both real estate sector and region. In 1998, 2000 and 2011, Lee and Byrne published three papers about region versus sector diversification in real estate. Their work is one of the fundamental contributions to the field. Lee and Byrne (1998) examine diversification using mean absolute deviation in the U.K., comparing numerous efficient frontiers using annual returns from 392 locations in the IPD Key Centres report (1981 to 1997). They concentrate on sector diversification using a mean absolute deviation model against three types of regional portfolios. In line with previous results, they found sector portfolios dominate regional portfolios. Byrne and Lee (2000) investigate risk reduction achieved across sectors and regions in the U.K., finding the greatest percentage reduction in total risk from naïve diversification across the three sectors (retail, office and industrial) and four regions (London, the South East, the South West and the North) occurred within regional portfolios spread across the three sectors. In contrast, sector portfolios spread across the four regions showed only minor

[10] A related issue is diversification by both sector and region.

risk reductions, with the office sector showing the worst performance. They explain this with correlations lower within a region than across sectors. Byrne and Lee (2011) study whether it is more advantageous to diversify by sector or region in terms of risk reduction using a recent database (1981 to 2007). They extend previous work by Byrne and Lee (1998). In line with previous research, the authors conclude property market sectors dominate regions. In these studies, Byrne and Lee (1998, 2011) apply the mean absolute deviation portfolio method from Konno (1988, 1990).[11] This approach overcomes numerous issues arising from the Markovitz approach.[12] Another study using a different approach but with comparable conclusions is Fisher and Liang (2000) in the U.S. market. They apply a dummy-variable method developed by Heston and Rouwenhorst (1994), dividing U.S. real estate into four sectors and four regions. Returns from real estate corresponded to variables that identify sector and regional affiliations. The authors argue that the NCREIF environment in the period 1977 to 1999 had better sector diversification than the regional one. Lee (2001) uses the same Heston and Rouwenhorst model for the U.K., decomposing total returns from the IPD Key Centres series into sector and regional influences. Results show sector effects account for most of the variation in property returns, accounting for more than three times the variability of real estate returns over regional factors. Results also demonstrate that two properties in the same sector are likely to be closer substitutes than two properties in the same region. Therefore, portfolio risk reduction was greater with a sector than a regional diversification. Using the same Heston and Rouwenhorst (1994) method, Andrew et al. (2003) found that for the U.K., the sector effect had a greater influence on property returns than regional factors using IPD annual data from 1981 to 2002, no matter the difference in specifications of sectors and regions.

Alternative to afore-mentioned papers that focus on traditional approaches, one can avoid classic portfolio theory or dummy variable method by using cluster techniques on return data from individual property markets. The method essentially holds in towns and cities, but large amounts of data are required. The objective is to cluster data by sector or region. If the regional dimension plays a role in return determination, property markets will cluster by location. This technique was used by

[11] Konno and Yamazaki (1991) demonstrate this procedure offers several useful properties in comparison to the Markowitz approach (see also Byrne and Lee (1997)).

[12] The traditional approach to investigating allocation problems has been to study correlations within real estate sectors and regions, and to use the classic mean-variance analysis from Markowitz. The primary criticism of a model based on correlation matrices is that only one dimension of diversification is considered. One must investigate the benefits of individual property risk. Another criticism is assumptions underlying the model such as the return distributions or investor utility functions. Assumption of return normality is generally invalid; see part 3 of the literature review.

Cullen (1991) on 5500 properties extracted from the IPD database in the U.K. Results suggest industrial assets are homogenous across the country, while retail spread more on ownership and lease terms and structure than on any regional basis. In contrast, the office sector displayed a distinct geographic structure, with city offices showing the greatest difference in comparison to the rest of the U.K. Two other groups of authors used this method: Hoesli et al. (1997) and Hamelink et al. (2000). They found similar results using quarterly IPD data (1977 to 1995). Office and industrial assets displayed strong geographic dimensions, but retail did not exhibit any geographic dimensions. The central London office market in particular, and especially the city office market, behaved differently from the rest of the U.K. Eichholtz et al. (1995) already obtained this same conclusion. More recently and using the same approach, Jackson and White (2005) found retail showed evidence of London clusters, but no evidence of any other references. It is important to point out that contrary to extant papers, Jackson and White use rental growth IPD data (1981 to 2000). In a recent paper, Hess and Ruggiero (2009) tried to determine whether recent changes in market conditions (subprime crisis) influenced the traditional market structure. Their finding was that the cluster technique works in the long-term, but is disturbed by extreme events such as abnormal risk levels.

Few studies deviate from this general trend of results. Among the few is Newell and Keng (2003) study of quarterly data in Australia for three sectors and three regions over the period 1995 to 2002 using the Heston and Rouwenhorst method. They show differences in sector and regional diversification were not as substantive as elsewhere, with regional diversification delivering slightly greater benefits than sector diversification. Importantly, they demonstrate both sector and region offer diversification benefits. Particularly relevant in this study is the more significant regional contribution to property diversification in Australia in comparison to the U.S. and U.K. Smith et al. (2005) also study diversification issues, starting with the assumption that real estate is essentially a local investment. They demonstrate that the major part of institutional real estate lays in the largest cities in the U.S., concluding the major markets are size-driven, not only geographic or sector driven.

C. Number of assets versus number of leases

In contemporary finance, portfolio diversification is studied by considering returns, variances and correlations of individual asset returns. In real estate, diversification uses another method, referred to as number of leases (or tenants).

Most academic literature suggests the number of assets required in a real estate portfolio is much larger than that held by most institutional portfolios. Brown (1988, 1991), Byrne and Lee (2000) and Callender et al. (2007) discuss this point. Real estate assets are indivisible by nature; holding a large number of assets requires a large amount of capital. Brown (1988) demonstrates most risk reduction can be achieved with relatively small numbers of properties. The marginal decrease in risk diminished rapidly after 10 properties. Other papers obtain similar results. Jones, Lang Wootton[13] (1986) argue nearly all possible risk reductions were achieved after 20 properties. Barber (1991) found that the majority of diversification was achieved once 40 to 45 properties were held in a portfolio. All of these results demonstrate that high levels of reduction in the variability of portfolio returns are possible with modest property numbers. However, these findings are based on averaging results across many portfolios; any particular portfolio may have a much higher level of risk. Many papers concentrate on portfolio return volatility versus diversification because reducing portfolio return variability does not mean good diversification. Brown (1988) shows that good diversification requires 200 assets to replicate the market at a 95% level. Brown's observation derives from a paradox: real estate assets display low correlation; a low number of assets allow rapid risk reductions, but because the market accounts for only a small part of the return for each asset, a large portfolio is required to obtain good diversification. Cullen (1991) and Byrne and Lee (2001) obtain 400 to 500 assets to achieve an acceptable level of diversification.[14] Callender at al. (2007) confirm previous results, finding that 30 to 50 properties are sufficient to achieve a high level of risk reduction, but full diversification requires very large portfolios. Other papers seek explanations to the small numbers of assets held by investors in comparison to academicians' recommendations. For example, Lee (2005) argues investors are rational holding small portfolios because marginal benefits of diversification are poor.

Regardless of portfolio size, various methods of portfolio construction may offer more efficient diversification strategies. In the context of multi-let properties or in

[13] Former name of Jones Lang Lasalle

[14] See Devaney and Lee (2005), who demonstrate benefits of increasing the number of properties to decrease downside risk.

portfolios containing a large number of assets (and possibly a larger number of tenant), one of the most important objectives of diversification is to reduce risk of income shortfalls. Mitchell (2012) argues real estate assets behave like the market but suggests lease events are the most important source of specific risk.[15] Income shortfall is central when considering loan covenants and ratios, and evokes the question of how many leases are necessary to decrease income shortfall risk. The number of leases sufficient to decrease risk of income shortfall has long been of concern to academicians and practitioners because income is the critical variable in most circumstance.[16] Mitchell (2012) considers the variability of returns based on income returns, not only total returns as is traditionally the case in much research. The approach examines income shortfall during the holding period of the asset or portfolio. Also focusing on cash flows, Robinson (2012) uses simulation techniques to isolate the impact of variables such as void periods, free rent and lease lengths. Simulation techniques have long been used to examine options embedded in leases such as break clauses, turnover and renewals (Booth and Walsh 2001a, 2001b; Amede-Manesme et al. 2012). Robinson (op. cit.) focuses on multi-let industrial properties, and concentrates not on valuation of leases, but on changes of cash flow. This research considers the number of leases required in a portfolio of multi-let assets to obtain the correct level of diversification, and the study uses Monte Carlo simulation to address the question. Robinson finds that 240 leases are the minimum to diversify a portfolio. Literature focusing on leases in the context of portfolio management and income is sparse, and largely influenced our choice to work in this area.

D. Inflation[17] hedging

Despite recent focus on commodities and gold, real estate remains an excellent inflation-hedging tool. Properties are a large part of individual expense, and their prices influence movements of inflation measures. Similar to other real assets, values of physical real estate relate positively to inflation. Particularly, underlying materials (concrete and steel) used for construction link to inflation. Contrarily, inflationary

[15] This thesis is written in this sense.

[16] For leveraged portfolios, expected income is more important since the lender had an interest in various debt ratios and loan covenants.

[17] Inflation is a measure of changes in the cost of living. Inflation is measured using a weighted basket of goods and observing price changes. In practice, there are many difficulties involved in measuring inflation. For this reason, inflation measurement is a conflicting measure among economists. We do not solve the debate here, but underline the issue.

pressures affect numerous financial assets like stocks and bonds adversely. In this section, literature and theories on inflation-hedging characteristics of commercial real estate (and briefly for housing) are discussed. There exists no consensus across countries concerning results and properties, but general results suggest direct real estate investment provides a hedge against inflation in the short and long terms, while securitized real estate investment affects only the long-term. Studies do not agree on this point, and we do not reopen the debate here.

For listed real estate, Hoesli et al. (2008) found there is some inflation protection in investing in indirect (and direct) real estate assets in the U.S. and U.K. Chatrath and Liang (1998) analyze the U.S. securitized sector and demonstrate some long-term but no short-term inflation-hedging properties. Using Fama and Schwert's (1977) model, Bahram et al. (2004) demonstrate REITs offer poor inflation-hedging properties in the long-term (evidence suggests REITs are coupled with the stock market).

The inflation-hedging capabilities of direct real estate investment are examined in real estate literature with mixed results. Hoesli et al. (1997) demonstrate in the U.K. that real estate is a good short-term inflation-hedging tool in comparison to bonds, but found stocks are a better inflation-hedging tool than real estate. In addition to this previous research (1997 focusing on direct real estate and 2006 focusing on both types), Hoesli et al. (2008) consider the short-term, inflation-hedging properties of U.K. real estate in comparison to other U.K. investments. The authors conclude real estate has poorer short-term hedging properties than stocks, but better than bonds. Simultaneously, Liu et al. (1997) obtained similar results, demonstrating stocks provide a better hedge against inflation than direct real estate in some countries and comparable hedging in others.

On inflation-hedging characteristics, two studies are particularly interesting. Sing and Low (2000) test inflation-hedging characteristics of real estate in Singapore, arguing industrial property is the most effective hedge against both expected and unexpected inflation (their work differentiates properties sectors). Bond and Seiler (1998) focus on the housing sector and suggest residential properties in the U.S. provide inflation-hedging capabilities. They use regression for the period 1969 to 1994.

Little research deviates from this trend of inflation-hedging properties of real estate. Stevenson and Murray (1999)[18] use causality tests to show real estate led inflation for Irish real estate, but did not find evidence that real estate is an inflation

[18] See Stevenson (2000) for interesting comments concerning real estate diversification and inflation-hedging using indices.

hedge. Glascock et al. (2002) conclude a non-significant correlation between inflation and real estate returns. It is important to note that most of the studies presented here do not differentiate property sectors: offices, residential properties and retail.

E. Optimal debt level

Over the last thirty years, use of debt increased substantially, with a peak in the years 2005 to 2007. Highly leveraged real estate funds, their impact on the economy and the risk undertaken by this kind of fund have been the subject of many publications. Today, the trend is driven by low-leveraged funds, and the new problem is more determining the optimal level of debt than maximizing debt.

Literature on debt does not provide a practical guide on how or why[19] to use leverage, and what is the optimal debt level for a fund or property. The widely accepted and taught theory comes from Modigliani and Miller (1958), who demonstrate company value does not depend on debt structure or level unless the debt provides tax benefits; tax advantage is the only reason for debt.

Smith and Boyd (1998) support Modigliani and Miller by analyzing the impact of debt on a mixed-asset portfolio both with and without leverage, and demonstrate an optimal debt level of between 20% and 75%.

In real estate context, Cannaday and Yang (1996) and McDonald (1999) show the existence of an optimal debt level for a real estate fund. These papers demonstrate optimal debt increases with tax rate. Tyrell and Bostwick (2005) support optimal leverage by considering rises in interest rates when debt increases. More recently, van der Spek and Hoorenman (2011) argue 40% is the maximum debt level, offering 3 reasons: high cost of distress, asymmetric performance fees, and impact of interest rates. Optimal debt has long been questioned by academicians, and the debate continues. Results and trends change widely with economic environments (and also with investor risk aversion).

F. Note on real estate derivatives

A derivative asset is an asset whose value derives from the value of another. The most salient are those associated with stocks and interest rates. Recent

[19] Anson and Hudson-Wilson (2003) list advantages and disadvantages of leverage, highlighting the stop-loss quality of debt. They discuss better diversification offered by leverage.

developments in derivative products have been made on the property market: derivatives contracts written on real estate performance indices such as the IPD index in the U.K. or the NCREIF index in the United States. In theory, derivatives provide important benefits to real estate investment management. In practice, many real estate derivatives emerge, and investor appetite is large, but at the time this thesis is written (2012), no market or index has taken a leadership position or created consensus. Direct real estate derivative products are essentially futures and swap contracts. Generally, no cash is exchanged at the time of the trade. However, an amount called the notional is traded (fictively: no flow), and then cash is exchanged according to performance of the real estate index. Investors can either buy or sell the real estate index, taking a long or short position.

The derivatives market has not yet caught on, but in theory, this market could greatly improve the efficiency of the real estate investment market. Real estate derivatives should be preferred to insurance-type contracts because of their direct settlement and liquidity. Case et al. (1993) point out liquidity can only be obtained if banks participate more actively in real estate index futures and options.

Academic research on real estate derivatives are relatively recent, and focus generally on indices. Hinkelman and Swidler (2008) show that housing price risk cannot be hedged with existing derivatives on commodities and financial indices. This comes from the incompleteness of the property market. Academicians agree on one point: it is unlikely individuals will use property derivatives to hedge house prices. Property derivatives are therefore for institutional investors because banks and other investors have an interest in hedging portfolios of loans based on property prices or hedging this illiquid investment. This topic is the primary area of research developed by Case and Shiller (1996) and Fisher (2005),[20] among others. Shiller (2008) provides a fundamental study in which he highlights barriers that need to be passed for establishment of a real estate derivatives market. Fabozzi et al. (2010) survey instruments available for real estate risk management, discussing limitations of this kind of product.

To complete this note on real estate derivatives, Geltner and Fisher (2007) focus on commercial real estate derivatives, proposing a review of key characteristics of property price indices (particularly differences between appraisal and transaction-based indices) and of all derivatives theories in light of commercial property derivatives. They examine derivatives pricing from both classic arbitrage and broader equilibrium

[20] In the French market, Gouriéroux and Laferrère (2009) published research on hedonic housing prices indices, highlighting growing demand for reliable indices and proposing a new index unexplored in light of real estate derivatives.

perspectives. Their paper is a reference for commercial property derivatives since many points are addressed (equilibrium pricing, risk, index knowledge, difference between real estate index and underlying markets).

Improvement in derivatives markets is fundamental for the real estate industry. It may improve efficiency in spot markets and price discovery, and it may remove barriers arising from transaction costs. It can also attract new investors to this asset class, especially those interested in the low correlation of real estate with other asset classes, and liquidity.

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In sum, these studies demonstrate that for a real estate fund manager, the first level of top-down analysis is global allocation to a portfolio because this weighting offers the greatest potential for risk reductions. Many papers show that real estate provides diversification benefits in a portfolio containing stocks and bonds. Subdivided allocations show superiority of a sector diversification strategy over the conventional regional approach (albeit without consensus). This led professionals and academicians to ask *what is a region*, and there is still research to be done on this topic. Number of leases provides diversification benefits, and is of interest to investors. In addition, debt and inflation hedging are unsolved issues; further research is needed. Beside these, the effect of break options and lease structures on portfolio management and/or on markets is an area that has not been researched sufficiently. Therefore, there are still large gaps in portfolio management and allocation literature concerning real estate.

II. Lease structure

Evolution of the commercial properties market in the 1990s was accompanied by a market evolution in leasing and occupational practices, with an abundance of short leases and break clauses. This created various investment characteristics for investors. In this section, we present leasing terms, characteristics and types, and introduce literature on the topic. We largely rely on Geltner et al. (2006)

A. Presentation of leases

The operating cash flow on which the value of commercial properties is based derives from the space market. More directly, leases moderate this operating cash flow, at least according to revenue. For residential properties, leases are short-term, typically a year.[21] In most other types of commercial property, long-term leases of various types are the norm. In all cases, the nature of leases and the major considerations in leasing strategy are key elements for management of commercial properties and are important determinants of investment performance and value of such assets. Without being comprehensive in treating commercial property leases, we introduce some basic terms and strategic considerations important from an economic perspective regarding commercial property leases and leasing. A number of characteristics influence the value of a property, and one of the most fundamental is the lease (location is first).

A lease involves a contract between a holder of property rights and a consumer or user holding at least some of those rights, covering a specified period. The property owner is the landlord. Generally, leases give possession and use rights but not development or redevelopment rights (an exception to this may occur in the case of very long leases). The holder of the lease is called the tenant, and the price of the lease is called the rent, typically paid periodically. In addition to possessory rights, the lease specifies other rights and duties on the part of the tenant and landlord. We do not expand here on lease law. Lease law is an extensive branch of the law, and commercial leases are usually complex legal documents. In addition, lease laws are country-specific.

Commercial property leases vary in how building expenses are treated. Some leases require the landlord to pay these expenses, some leases require the tenant to pay, and others provide a share of operating expense. The landlord has the advantage of giving the tenant operating expenses, offering protection against inflation of those expenses. It allows therefore a transfer of building operating risk to the tenant. There are numerous kinds of leases, the most popular of which include:

- Gross lease. The landlord pays operating expenses. This kind of lease is also called a full-service lease (such as Regus or Multiburo).
- Net lease. The tenant is responsible for operating expenses. Sometimes this kind of lease is referred to as triple net; the

[21] The exception is France among others.

landlord covers some expenses such as property manager fees and some taxes.

- Hybrid lease. Combines aspects of both gross and net leases. The tenant is responsible for some expenses and the landlord pays the remainder.

1. Type of rent change in leases

One of the most important elements of long-term commercial leases of interest to investors is the way rent changes over time during the term of the lease. The reason for rent change is to protect the landlord from inflation and price changes. Rent changes can take various form: graduated when rent increases from a specified amount on a specified date as stipulated in the lease; indexed when rent adjusts according to a publicly published index (generally the CPI or a percentage of the CPI, but can be specific such as cost of construction in France or a health index in Belgium); reevaluated when rent changes on a specified date given an appraisal of rent (sometimes rent reviews take the form of upward-only adjustments while other leases allow adjustment to be in either direction).

It is important to note that rent can also be indexed to revenues or profit in the case of a retail space.

2. Break options (or break clauses)

The time covered by a lease can have implications for a landlord. A landlord generally prefers a long-term lease. In some cases, leases contain options.[22] Leases often provide explicit options to both tenant and landlord (dual option) or to just one (asymmetric option). An option is a right without obligation to obtain some benefit. Sometimes an option requires payment of an additional sum (the premium) to exercise the option. Options provide flexibility to the holder, and the most common include:

- Renewal option: the tenant has the right to renew the space at the end of the lease. Sometimes this option specifies future rent, similar to a right of first refusal.
- Break option or break clause: the holder of the option has the right to terminate the lease prior to the term of the lease

[22] These options are alternatively called break-options or break-clause options.

within a specified notice period. The option may be written either for the tenant or the landlord. Break clauses can take many forms; there is no universal form or type of break clause. Issues related to drafting, timing, beneficiaries, penalties and frequency vary. This point is exacerbated further by the diversity of break clauses and letting circumstances, which produce diversity for financial implications.

- Expansion option: this option takes the form of a right of first refusal for the tenant for a space adjacent to leased space. The option can also take the form of offering the tenant the right to rent space at a specified price during a specified period. These break-options are found particularly in continental Europe leases, and motivated this thesis.

B. Literature on lease structure

Numerous papers examine the effect of commercial property leases on market structures and investor behavior. The same authors or group of authors largely write these papers. The implications for investment strategy of a short or long lease, pricing of leased property, cost of vacancy, probability of vacancy and global influences of all these factors on market rental value volatility have been the subjects of many papers. From our viewpoint, this is not sufficient, and many points have not been questioned. This is due primarily to the specificity of lease structures and regulations among countries. Leases are country-specific, and comparison is difficult due to discrepancies in lease lengths, indexations, options, renewals etc. Particularly, pricing of a lease structure is a risk factor, more an asset-specific risk factor than market or portfolio risk, and therefore it is a more difficult risk to study. However, lack of reliable data on patterns arising from lease terminations and lack of reliable time series of historic property performance limit the efficacy of research in the field. We present here primarily contributions to the field. We first discuss research on the pricing of lease structures, and then research on options applied to leases.

In research of pricing lease terms and structure variation, the traditional assumption of lease pricing models is that real estate investors extract the same value from a property regardless of lease structure. One of the most significant papers on lease pricing is Grenadier's (1995) analysis of a range of lease options. Grenadier studies parallelism of varying lease lengths with bonds that have comparable maturities. He determines rent equilibrium for any lease length and the term structure of lease rates analogous to a term structure of interest rates. Grenadier proposes numerous assumptions concerning patterns of lease lengths given various market conditions. Following the method used for the term structure of interest rates, he describes three possible term structure shapes—downward-sloping, upward-sloping and single-humped—driven by landlord expectations. He also demonstrates that short lease terms do not mechanically produce higher rent, and justifies why some owners, under certain market conditions, prefer short leases or fixed rent.[23] However, his work is based on many hypotheses. Among them, the model does not consider transaction costs, vacancies, taxes and landlord behaviors. Other contributions to the field come from McAllister and Tarbert (1999), McAllister (2001), Mcallister and Roarty (1998, 1999), McAllister (2008) and McAllister and Fuerst (2008). They examine growth of short leases (or the growth of break clauses) and implications of both shorter leases and break options in commercial property investments. McAllister identifies many critical variables influencing the effects of short leases on risk and return. In terms of valuation, no general rule can be applied to account for break-options. This was also the argument of Herd and Lizieri (1994), who found valuers tend to use ad hoc adjustments to reflect the effects of break clauses.[24] These authors highlight a strong lack of consistency among valuers. They build a simulation approach to account for the possibility that tenants exercise the option to leave. They also found evidence of inconsistency in application of yield adjustments as a remedy for the impact of break options on value. Their models fail to incorporate additional risks inherent in break clauses and short leases or the possibility of downward rent reviews or free rent.

Another field related to lease options raises considerable interest among researchers: the application of option pricing techniques to property investment and development decisions. The choice to vacate can be viewed like a typical option. The first work on this topic are those of Grenadier (1995), Ward (1997), Ward et al. (1998), Patel and Sing (1998), Rowland (1999) and Baum (2003). More recent research[25] on these topics come from Booth and Walsh (2001a, 2001b), Ambrose et al. (2002),

[23] If they expect market conditions to improve, they should regard short leases more favorably.
[24] This is due both to difficulty in assessing the effect of a break-option on the market (the effect may be different from one asset to another and from one location to another) and difficulty in finding comparable transactions to apply comparable models.
[25] These works concentrate on U.K. and U.S. real estate markets, but are interesting.

Hendershott and Ward (2003) and Stanton and Wallace (2009).[26] A paper by McCann and Ward (2004) is particularly appealing; they study optimal lease lengths from the tenant's viewpoint and question applicability of a term structure. Results suggest that from the tenant's perspective, the cost of space changes with the length of a lease independently from the term structure of rental rates. Occupancy value is a function of tenants' specifications such as legal costs, search costs, moving costs, double rent, repairing costs and opportunity costs. They develop a model that, given assumptions, prices the range of lease lengths for individual tenants. The conclusion is that business reasons dominate the term structure of rent, not financials. It is clear that researchers are unable to identify an expected term structure of rent (Clapham et al. 2006; Bond et al. 2008).

Some limitations appear in these works, the source of which is differences between real estate assets and other products. In a typical option, an investor acquires the right to buy (call option) or sell (put option) an underlying asset before or on a pre-defined date. In real estate, our concern is the option to vacate conceded to a tenant. It is possible to find an analogy with European options and, more precisely, a European put option where the tenant has the right to sell the lease (to vacate) on an agreed date. The option's value is a function of movement in the price of the underlying asset. In theory, the higher the volatility of the underlying asset, the higher the option's price. In the option-pricing model, the key variable is volatility due primarily to the fact that volatility is the only factor estimated (historically or implied in a volatility model). Even in deep real estate markets, volatility is misestimated. This derives from a well-known real estate issue discussed already in this thesis: lack of reliable historic time series and/or adequate transaction data. Many academicians (Hodges 1990; Hamill et al. 2006, 2008, 2011) have long highlighted this point on volatility and options. Today, it is still an issue for lease options pricing.

A further field that have raised a large amount of academic research is the financial and economic theories that suggest lease and debt are substitutes. An increase in one leads to a decrease in the other according to a potential substitution coefficient. The basic idea behind this comes from the analogy between cash flows from the lease obligations and cash flows from debt cash flows. One of the most fundamental researches in this field[27] is Ang and Peterson (1984) who conduct a series of empirical tests. The authors demonstrate that leases and debt are complements instead of substitutes. This idea has been the primary one in the literature since today.

[26] Clapham and Gunnelin (2003) discuss the limitations of options theory in property options.

[27] See also Franks and Hodges (1978) and Mukherjee (1991). In addition to that, Myers et al. (1976) derived a formula for evaluating financial lease contract.

Recently, this filed has been the subject of some papers. Among others, Eisfeldt and Rampini (2009) studies the financing role of leasing. They argue that the benefit of leasing is that repossession of a leased asset is easier than foreclosure on the collateral of a secured loan. This suggests that leasing preserves capital.

An issue appearing in all of these studies is ignorance of the impact of heterogeneity, even though this shortcut is mentioned systematically. For example, decisions to move are driven by rational factors, but more on a volunteer basis than from management.[28] In addition, landlord behavior is unclear and largely dependent on strategy. Researchers such as Miceli and Sirmans (1999) demonstrate that landlords attempt to minimize vacancy and turnover costs by offering discounts to long-term tenants. Landlords may minimize the number of possible break-options conceded to tenants by giving discounts or free rent.

Leases, therefore, are important factors in property returns and for the market. They influence volatility and distribution because they play a role in global market risk. In that sense, we discuss research of returns distribution in the next section.

III. Distribution of returns

Many practitioners optimize portfolio risk and diversification using the traditional Markovitz (1952, 1968) mean-variance model and its corollary, the Capital Asset Pricing Model, developed by Sharpe (1964), adapting the model to commercial real estate. Literature related to these models in real estate was presented in the first section of this literature review. The relevancy of the modern portfolio theory model is not discussed here, but we raise strong doubts concerning assumptions, and review literature on the topic. Nevertheless, the traditional approach is justified if the assets are both traded in frictionless liquid markets and asset returns are independently and identically distributed over time. In this context, the solution to the long-term asset allocation dilemma is similar to the short-term one. Nobel Samuelson (1969) and Merton (1969) demonstrate this. In real estate, assumptions underlying the classic results of Samuelson and Merton are likely violated.[29] It is widely assumed that returns

[28] There exists heterogeneity in the probability of exercise in the real estate market. In a typical European option, the rational investor always exercises an option if it is in the money. In real estate, each break option is unique in terms of structure of the option and tenant attitudes to exercise.

[29] Transaction costs and lack of liquidity are obvious market frictions.

from real estate are not normal, and thus not independently and identically distributed. Many academicians have started to examine real estate return distributions. However, estimating returns over long periods is an arduous task in real estate, particularly in direct real estate. Many empirical tests in the real estate literature support how important it is to assume non-normality of property returns. Non-normality of real estate returns has soon intrigued academicians, and has been a strong debate in the literature. Young et al. (1995, 2006, 2008) and Lee et al. (1997, 2002, 2006) published numerous papers on the topic. One of the key points in direct real estate is lack of data, and all traditional models require high frequencies of data for correct estimation of the distribution. The papers discussed below concern U.K. and U.S. data, once more based on availability. We share the following presentation between direct and indirect real estate.

A. Direct real estate

In direct real estate, the frequency and reliability of data is particularly critical. Nevertheless, many researchers publish papers on return distributions. Among the first papers on the topic, Miles and McCue (1984) or Hartzell et al. (1986) found evidence of non-normality by focusing on skewness and kurtosis coefficients. Similarly, Myer and Webb (1994) confirm evidence of non-normality using kurtosis, and demonstrate autocorrelation in private real estate returns. The most fundamental empirical research on the Gaussian distribution assumption comes from Young and Graff (1995). Using the NCREIF database (1980 to 1992), they characterize non-normality of U.S. institutional private real estate distributions, demonstrating how an infinite variance model provides better descriptions of distributions of returns in the property sector. They particularly discuss how the diminution of risk from diversification is less effective in the non-normal context. Byrne and Lee (1997) examine quarterly returns for sectors and regions using the NCREIF (1983 to 1994). The Jarque Bera test rejects normality for 10 of 16 sub-sectors, consistent with earlier findings because they find positive kurtosis and demonstrate how real estate returns exhibit negative skewness. They point out how portfolio allocation, using volatility as a risk measurement, is unsuitable. Graff et al. (1997) examine the distribution characteristics of Australian real estate based on the Property Council of Australia's Performance Index, a short time series (1984 to 1996). The study highlights problems of a direct real estate database (low frequency of data, small number of assets, valuation). They conclude an abnormal shape of the distribution for Australian property returns. The shape of the distribution is nearly the same as the one for the United States. Australian real estate investment is therefore heteroscedatic. In terms of portfolio management, this shows that models that rely on finite variance statistics are inefficient in the Australian real estate context. Lizieri and Ward (2000) also examine the distribution of returns in the commercial real estate market, focusing on direct real estate in the U.K. They concentrate on office, retail and industrial properties to discriminate commercial real estate from private residential markets. They highlight the non-normality of property returns based on the IPD monthly index database. The logistic distribution is only one of possible distributions. Implications of distributions for asset allocation are examined. Lizieri and Ward contradicted two received ideas. First, they do not attribute aberrant behavior of real estate returns to appraisal or smoothing issues. Second, they suggest real estate returns produce better results in less frequent database, returns being easier to model. According to the authors, these points come from real estate characteristics such as heterogeneity, indivisibility, valuations versus transactions data or high transaction costs. Young et al. (2006) support previous results, emphasizing how unsafe it is to assume normality in real estate returns using annual IPD data (1981 to 2003). They analyze subsectors and reject the normality assumption at the sub-sector level. Analysis demonstrates how the skewness coefficient and the magnitude of real estate assets change over time. More recently, Young (2008) extends previous results to a more recent database based on NCREIF (1980 to 2003). In this updated empirical analysis, the authors find evidence of non-normality, and show how differences among property types appear salient. Real estate risk is shown to be heteroscedastic. Once again, the paper demonstrates asset diversification is less effective than expected. Outside English-speaking countries, Richter et al. (2011) question the normality of German property returns based on IPD Database. Using quantile-based estimation, they examine distributions of income, capital growth and total returns, revealing that the assumption of normality in return distributions can be rejected for all subsamples of all property types.

B. Indirect real estate

Many studies concentrate on indirect real estate return distributions. For listed real estate, more data are available, and it is easier to determine a distribution. These returns provide evidence of skewed, peaked distributions and fat tails. The distribution of listed real estate has long been questioned in the literature, and many papers concerning nearly all countries are available. We only present a general survey of this literature because we do not focus on listed real estate (REITs) in this thesis.

A paper by Lizieri and Satchell (1997) examines the distribution of monthly property company (as opposed to REITs) returns in the U.K. between 1972 and 1992, demonstrating strong evidence for non-normality. The Jarque-Bera test rejects the hypothesis of normality, and returns exhibited positive skewness and kurtosis coefficients and were fat-tailed.

For purely listed real estate (REITs), Seiler et al. (1999) investigate the return distributions of equity real estate investment trusts (EREITs) from a quarterly database (1986 to 1996). In spite of the size of database, the Kolmogorov-Smirnov, Shapiro-Wilks and Lilliefors tests rejected normality. They conduct the study on a sector basis and expose results sector by sector. For example, office REITs returns were less normal while industrial REITs returns did not reject the tests. They highlight that offices exhibit high volatility and are skewed positively. More recent research confirms these results. Lizieri et al. (2007) identify and model excess kurtosis in REITs returns. Cotter and Stevenson (2007) demonstrate for REITs returns that conditional heteroskedasticity and volatility can be modeled with many GARCH models. Cotter and Roll (2011) also provide elements for REITs return characteristics. They compare it to the S&P500, and REITs exhibited lower market risk (beta) but comparable total volatility. Return distributions displayed negative skewness and very high kurtosis, implying REITs returns are non-normal.

Another classic question when considering listed real estate returns remains unsolved; are REITs stock or property? This question was discussed in the portfolio allocation part of the literature review, and has been the subject of numerous studies. Today, no consensus emerges, leaving considerable work for academicians. Here, we mention studies that discuss it in light of distributions. Many researchers note that REIT and property companies sharing returns have much closer contemporaneous correlations with the stock market than with the underlying real estate market. These results hold even when researchers correct the distribution for appraisal smoothing in the direct property market and for gearing (leverage) in the indirect market as demonstrated by Barkham and Geltner (1995). We highlight the fact that information about REITs return distributions does not necessarily offer information on direct real estate returns. Interest in this field is reported to MacKinnon and Al Zaman (2009) and Hoesli and Oikarinen (2012).

Studies suggest that the normality of real estate asset returns is sensitive to data frequency. Using conventional statistical approaches on monthly data, Lee (2002) demonstrates positive skewness in nearly all markets. Booth et al. (2002) show evidence of kurtosis, with tails particularly fat. Contrarily, Maurer et al. (2004) find no evidence of either skewness or kurtosis using quarterly data from the U.K. Lizieri and Ward (2000) partially contradict Maurer et al. (op. cit); monthly returns were non-normal, and normality was not rejected at the sub-sector level for quarterly returns in 2 of 5 subsectors. As discussed, Young et al. (2006) used the IPD U.K. annual data (1981 to 2003), rejecting the normality of distributions of real estate returns. There exists no consensus on the optimum data frequency, and the debate on normality continues. As we show in part three of this thesis concerning Value at Risk using Cornish-Fisher expansion, the assumption of normality collapsed after the 2007 subprime crisis (lack of recent researches from the best of our knowledge).

Few studies in real estate consider non-normality in portfolio management and allocation, and in risk management. It is partially the objective of this thesis to consider non-normality, particularly in the case of risk management. Non-normality has huge effects on Value at Risk assessment and more generally on risk management. This is why we concentrate part of this thesis on Value at Risk.

IV. Value at Risk

A. Introduction and definition[30]

Risk measurements have changed remarkably since Markovitz (1952) developed his theory in the 1950s. At that time, standard deviation was the risk measure of an efficient portfolio, but this measure was not relevant for only one security. For a single security, risk is computed using covariance between the security and the market. A security's standard deviation is comprised of risk that can be mitigated by diversification and risk that cannot be diversified, yet only the risk that cannot be diversified is remunerated. Risk theories that followed concentrated primarily on factors that determine security risk and capital market equilibrium. When considering a portfolio comprised of N securities, Markowitz's model requires the estimation of N variances and $\frac{N^2 - N}{2}$ covariances. When N becomes large, estimation of the variance-covariance matrix becomes arduous, and the possibilities for errors increase, which can lead to misleading decision. During the 1960s, Sharpe (1964) developed the Capital

[30] This introduction largely takes a leaf out of Racicot et al. (2006).

Asset Pricing Model, a mono-factor model that considers covariance between a security and a market as the only risk factor. Risk is represented by the beta (β) and is called the systematic risk, but it cannot be mitigated by diversification. Contrarily, specific (non-systematic) risk inherent to a company can be mitigated by diversification. Then, Ross (1976) developed the Asset Pricing Theory in the 1970s, a multifactor model that identifies the multidimensional effects of risk. One of the weaknesses of this model is that it does not explain factors that determine security returns.

Value at Risk did not appear before the late 1980s. In 1987, the stock market crashed, a triggering event for development of new risk measures. This was the first major financial crisis in which practitioners and academicians were afraid of global bankruptcy of the entire system. The crash was so improbable given standard statistics models, that many quantitative analysts cast doubts and began to question the models. Many academicians claimed the crisis would recur and called for reconsidering the models. Considering extreme events became necessary. Limitations of traditional risk measures were recognized, and measuring risk of falling asset values was becoming urgent. The necessity of relying on a risk measurement that considers the entire distribution of returns of a portfolio was obvious. Throughout the 1990s, a new risk measurement was developed: Value at Risk (VaR).[31]

VaR was developed and adopted by practitioners and regulators. Jorion (2006) defines VaR as "a method of assessing risk that uses standard statistical techniques used routinely in other technical fields. Loosely, VaR summarizes the worst look over a target horizon that will not be exceeded with a given level of confidence." In financial risk management, VaR is a risk measure of loss on a portfolio of financial assets. For a given portfolio, probability and time horizon, VaR is a threshold value such that the probability that the mark-to-market loss on the portfolio over the given period exceeds this value (assuming no trading in the portfolio) is the probability level. For portfolio value at time t, V_t, for one period and for threshold α, this can be expressed as:

$$\forall t, \mathbf{P}_t \left[\left(V_{t+1} - V_t \right) + VaR_{t,\alpha} < 0 \right] = \alpha$$

Considering a centered and reduced random variable U with cumulative distribution function F_U and u_α, the quantile at threshold α, we have:[32]

$$VaR_\alpha(U) = -\sup\left\{ u \mid F_U(u) < \alpha \right\} = -u_\alpha$$

[31] For this thesis, VaR is assumed to be computed for a static portfolio, with no change in its structure and no trading or arbitrage.

[32] Some choose to count effective loss positively: $VaR_\alpha(U) = \inf\left\{ u \mid F_U(u) > \alpha \right\}$, which is equal to u_α if the distribution is symmetric.

This measure is becoming increasingly popular to value risk of institutional and individual portfolios. VaR is an easy to understand method for quantifying market risk. Today, VaR is used by many regulators as a risk measurement reference for Basel I, Basel II, Solvency II and NAIC, among others.

Worldwide adoption of the Basel II Accord in 1999 and near completion in 2012 (Basel III must be applied for 2019) offers further motivation for VaR. The Basle committee required that banks compute their VaR periodically and maintain sufficient capital to pay eventual losses projected by VaR. Unfortunately, there is no single measure of VaR because volatility, a fundamental component, is latent. Banks must use many VaR models to compute a range of prospective losses. More recently, the Solvency II regulation for insurers in Europe proposed VaR as a reference measure to determine required capital. Like Basel II, it proposes either use of a standard model or an internal model. The standard model for real estate VaR leads to required capital of 25% for real estate investments. This calculation was made based on IPD U.K., all properties total return index (this index is a reliable commercial monthly index in Europe).[33] The committee recognizes the non-normality of real estate returns, but did not try to estimate real estate required capital. It recommends an historic VaR of 25%, leaving further research for internal model.

VaR computations are complex because distributions of returns are generally unknown. The primary uses of VaR in finance are risk management, risk analysis, financial control, financial reporting and computing regulatory capital. Methods and risk measurements such as stress testing, expected shortfalls, and tail VaR have become more popular because they focus particularly on expected severity of failure. VaR is slowly replacing standard deviation (or volatility) as the most widely used measure of risk[34] because of the need for a single risk measure for calculation of capital adequacy limits for financial institutions such as banks and insurers. VaR allows regulators and bank managers to put a single number on a predefined worst-case scenario, at a predefined confidence level.

The three traditional methods of calculating VaR are the historic, the parametric and the Monte Carlo methods.

- The historical method involves taking empirical profit and loss history and ordering them, assuming history repeats. The main

[33] From the Solvency II report, referenced as CEIOPS-DOC-40/09 (2009):
- 3.160: “One of the most challenging factors of this specific calibration is the lack of long time series across most European markets.”
- 3.169 & 3.171: “All distributions of property returns are characterized by long left fat-tails and excess kurtosis signifying disparity from normal distribution.”
[34] This point is subject to debate.

benefit of the method is that it does not require assumptions concerning the nature of the distribution of returns. A drawback is that this method assumes the shape of future returns will be the same as those in the past. To make this approach statistically reliable, one needs to ensure that a sufficient number of observations are available and they are representative of all possible states for the portfolio. Data must incorporate observations from both bull and bear markets. In real estate, since we rarely have enough history (more generally in almost all non-listed markets), the method is not as accurate as either the parametric or simulation methods.

- The parametric method (sometimes called variance-covariance method) requires an assumption concerning the statistical distribution (normal, log-normal etc.) from which the data are drawn. Parametric approaches are comparable to fitting curves through data and then reading off VaR from the fitted curve (unfortunately, for many sophisticated models, analytical solutions do not exist). The parametric VaR is one of the more popular methods. The attraction of parametric or analytic VaR is that relatively little information is needed to compute it. A weakness is that the distribution chosen may not reflect all possible states of the market and may under or overestimate risk. This problem is particularly acute when using VaR to assess risk of asymmetric distributions (left skewed distribution or one for a particular portfolio containing options). In such cases, higher statistical moments of skewness and kurtosis, which contribute to more extreme losses (fat tails), need to be considered. Although some level of statistical sophistication is necessary, parametric methods exist for a wide variety of distributions.
- The Monte Carlo approach has become increasingly popular in recent years, due to improvements in computer and software power. Monte Carlo methods rely on repeated random generations from a probability distribution of inputs that are used to compute model results. Simulation-based VaR generates many scenarios drawn from either a parametric assumption about the shape of the distribution (Monte Carlo) or re-

sampling (bootstrap) the empirical history and generating sufficient data to be statistically significant. VaR is deduced by reading the desired percentile as with the historic method.

After defining VaR and a presentation of the primary methods of computing VaR, we review the literature on the subject and introduce the sparse literature that concentrates on real estate VaR.

B. Literature review

VaR has been the subject of broad research among academicians. All methods proposed to compute VaR or a distribution quantile have been subject to academic research after development of VaR. A large portion of the literature focuses on ways to measure VaR. For example, Linsmeier and Pearson (2000), Duffie and Pan (1997) and Engle and Manganelli (1999) published general papers on measuring VaR. More specific research was published on primary methods, among them Monte Carlo simulation from Pritsker (1997), Johnson transformations from Zangari (1996a), Cornish-Fisher expansions from Zangari (1996b) and Fallon (1996), Solomon-Stephens approximation from Britton-Jones and Schaefer (1999), saddle-point approximations from Feuerverger and Wong (2000), Fourier-inversion from Frolov and Kitaev (1998) and extreme value theory from Longin (2000). Theoretical properties of VaR evaluation is reported by Artzner et al. (1999), Cvitanic and Karatzas (1999) and Wang (1999).

Other attention is given to VaR optimization for portfolio or risk reductions. Roy (1952), Telser (1967) and Kataoka (1963), who focus on portfolio constraints called safety-first approaches at the time, started these works. Emmer et al. (2001), Alexander and Baptista (2002, 2003), Embrechts et al. (1999) and Kast et al. (1999) extended research on VaR. Also, the work of Gourieroux et al. (2000) about the sensitivity of Value at Risk is appealing.[35]

In addition to all these methods, many works concentrate on the best methods to compute VaR or on the pros and cons of VaR. Jorion (1996) explains that the benefits of VaR should not mask its shortcomings because any VaR number is itself measured with error or estimation risk. Jorion recommends reporting a confidence interval with the VaR number to be more relevant. The author highlights that the

[35] the authors derive analytical expressions for and second derivatives of the VaR.

greatest advantage of VaR computation is to think globally and critically about risk. Pichler and Selitsch (1999) compare five VaR methods in the context of portfolios that include options using Johnson transformations, variance-covariance, and three Cornish-Fisher-approximations for the second, fourth and sixth orders. They conclude that the sixth-order Cornish-Fisher approximation is the best approach. Mina and Ulmer (1999) compare Johnson transformations, Fourier inversion, Cornish-Fisher approximations and Monte Carlo simulation. Their conclusions include the following: Johnson transformations are not a robust choice. Monte Carlo and Fourier inversion are robust, and Cornish-Fisher is extremely fast but less robust than the two previous approaches, particularly when the distribution is far from normal. Feuerverger and Wong (2000) focus on when to use Cornish-Fisher in comparison to Fourier inversion, saddle point methods, and Monte Carlo. The paper concludes with an extension of the method, which includes higher-order terms. Candelon et al. (2011) propose a method to backtest VaR models using a duration-based backtesting procedure derived from the GMM test introduced by Bontemps (2008). This allows testing distribution assumptions using an easy approach. Thus, the authors introduce a new way to backtest VaR estimation and forecasts. Other academicians such as Longin (2005) suggest taking interest in extreme events—and therefore extreme value theory—only when appraising extreme risk such as VaR.

The sub-additivity issue of VaR has recently[36] been the subject of a few papers, particularly after the subprime crisis in 2007 when the non-normality of returns was rediscovered. A description of the underlying issues and further references can be found in McNeil et al. (2005). Other research on the subject includes Rootzén and Klüppelberg (1999), Dhaene et al. (2009) and Daníelsson et al. (2012). Heyde et al. (2006) discuss robustness of risk measures, suggesting replacement of tail conditional expectations (sub-additive risk measure) with a more robust tail conditional median, which is simply VaR at a higher confidence level.

The debate concerning VaR relevancy is old and not the purpose of this thesis. We accept the choice of VaR by regulators without discussion, and try to develop a better VaR valuation model.

Research of risk focusing on direct real estate or unlisted vehicles is scarce in spite of increasing interest. Again, this is probably partially due to the paucity of available data. Either invest in listed real estate, real estate data is quoted daily and

[36] Artzner et al. (1999) discuss strengths and weaknesses of VaR as a risk measure.

sufficient data are available to compute VaR for a portfolio, or invest in direct real estate and deal with small datasets. This is particularly true in commercial real estate in which institutional investors largely invest, and where VaR computation is mandatory for capital adequacy. The real estate market is comparable to the private equity market where indices are built on smaller numbers of transactions. A real estate property index aggregates real estate market information to provide a representation of underlying performance. However, this is conducted monthly in the best of cases (U.K., U.S. and Australia), quarterly or semi-annually occasionally but generally yearly, largely linked to sector. The residential field, in which many transactions can be observed, exhibits a monthly index frequently. Contrarily, commercial real estate faces more difficulties to deliver indices with larger periodicities.

Research of VaR in real estate focuses on REITs or securitized real estate. VaR for listed real estate relies on methods for stocks or bonds. Zhou and Anderson (2012) concentrate on extreme risks and the behavior of REITs in abnormal market conditions, arguing no universal method can be recommended for VaR in listed real estate. They note estimation of risk for stocks and REITs requires disparate approaches. Cotter and Roll (2011) study REIT behavior over 40 years, highlighting non-normality of REIT returns. They compute VaR for the index following three methods that do not rely on Gaussian assumptions: Efficient Maximization algorithm, Generalized Pareto Distribution and a GARCH model. The authors highlight that reality is much worse than that depicted by an assumption of normally distributed returns. Liow (2008) uses extreme value theory to assess VaR dynamics of ten major securitized real estate markets. Extreme value theory allows the author to consider the stochastic behavior of the tail. Using this tool, extreme market risks are better assessed than with traditional standard deviation measures, and real estate forecasts are more accurate.

Literature focusing on VaR in the context of direct real estate investment is sparse. However, numerous studies concentrate on risk management and assessment in real estate. Booth et al. (2002) examine risk measurement and management for real estate portfolios, explaining that practical issues force real estate investors to treat them differently from other assets classes. They particularly espouse that direct real estate is an area for further research. The report is a complete review of the range of risk measures that can be used to measure real estate risk. It focuses on the difference between symmetric measures such as standard deviation and downside risk measures such as VaR. The authors conclude by recommending use of risk measures “that properly reflect their subjective risk preferences and do not use standard deviation of investment returns for the sake of simplicity. [...] Different risk measures measure different aspects of risk." Their work concentrates on all risk measures usable in real estate. They do not conclude with one perfect risk measure, but propose adapting risk measures to needs. Gordon and Wai Kuen Tse (2003) consider VaR a tool to measure leverage risk in a real estate portfolio. Debt in a real estate portfolio is a traditional issue in real estate finance. The paper demonstrates VaR allows better assessment of risk. Traditional risk-adjusted measures (Sharpe ratio, Treynor's and Jensen's alpha) suffer from the leverage paradox; leverage adds risk along with the potential for higher returns per unit of higher risk. Therefore, the risk/return ratio does not change noticeably and is not an accurate tool to measure risk inherent to debt. They conclude VaR is good tool for leverage risk. Farelly (2012) focuses on measuring risk of unlisted property funds using a forward-looking approach. Among other results, the author considers moments measures of orders greater than two (asymmetry considered) using Cornish-Fisher expansion.

Recommending other risk measures, Brown and Young (2011) focus on a new way to measure real estate investment risk. They refute the assumption of normally distributed returns that flaws forecasts and decisions, and the nature of risk and how it should be measured is discussed. Interestingly, VaR is not retained, and the expected shortfall is recommended more. The authors then focus on spectral measures, which represent their final recommendation.

Following these authors, we motivate the third part of this thesis with needs for better VaR assessment in direct real estate.

C. VaR limitations and Conditional VaR

VaR is not the maximum loss an investor can realize; it is the lowest loss at threshold α. Even if exact in theory, it works only within a specified confidence level. There is always a higher level of loss for a lower confidence level.[37] Despite its popularity among practitioners, regulators and academicians, VaR is subject to many criticisms. It has been controversial since it moved from trading desks into public awareness in 1994. A common complaint among academicians is that VaR is not sub-additive; VaR does not systematically satisfy the property of convexity as illustrated by Danielson et al. (2005). The VaR of a combined portfolio can be larger than the sum of the VaR of its components. This was demonstrated by Artzner et al. (1999). Except in some special cases (among which normal distribution), VaR does not satisfy

[37] Kerviel's event happened only once in Societé Générale history.

the sub-additivity requirement for mathematical coherence and for simple, practical use. Assessing plausible losses is difficult using VaR theory. Losses can be extremely large and sometimes impossible to determine once an investment goes beyond the VaR point. From a practitioner's viewpoint, VaR is more the level of losses at which an investor stops trying to imagine what worse can happen next.

VaR is a risk measure that only considers the probability of being below a threshold level; it does not consider values below this level or their averages. This is why other risk measures have been proposed. Among them, expected shortfall as defined by Acerbi et al. (2001) (also called conditional VaR: *CVaR*) or the TailVaR from Artzner et al. (1999). Applications of CVaR in real estate finance are nearly non-existent, and applications and research on the topic are currently in preparation.

Mentioned above, the objective of this thesis is not to discuss the quality of VaR as a risk estimator or its adequacy for risk-budgeting purposes. Regulators chose VaR for a required economic capital calculation, and its computation is now mandatory for all regulated practitioners. Therefore, academic research on the topic is now also mandatory.

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This literature review was designed as prose. The writing tries to avoid providing a list describing or summarizing one piece of literature after another. The literature review is organized into sections that present themes or identify trends, including relevant and irrelevant theories. It cites the most relevant published papers, and synthesizes and evaluates the papers in light of this study. We apologize for missing references and inaccuracies.

PART II – REAL ESTATE PORTFOLIO AND RISK MANAGEMENT

This part focuses on real estate portfolio and risk management, concentrating particularly on a topic that has not been the subject of sufficient academic or professional research: break options. Another title could have been *how break clauses can be considered in real estate portfolio management.* Break options are options embedded in leases in favor of the tenant, comparable to financial options. These options are a continental Europe lease specificities, but this kind of option is standardizing, and they are sometimes used in the U.K. or the U.S. (even if their use is in a state of infancy in these countries). The break options are one of the most important risk real estate finance must deal with and we introduce in this part a model that allows including them rationally in cash flows. This way, this real estate specific risk can be considered.

Real estate portfolio (or fund) management differs radically from managing equity, bond or mutual funds in that real estate managers are not only responsible for asset allocation, risk management and transaction supervision, but also managing execution of asset strategy. Unlike equity managers who research companies and manage probabilities that they can execute strategies, real estate managers must define and execute strategies at the property and portfolio levels. Therefore, real estate asset management - contrary to traditional asset management of stocks and bonds - requires experienced individuals who optimize the value of properties through superior transaction services, information and management execution. Real estate portfolio managers conduct asset allocation and asset selection not only by understanding fundamentals of the market and/or of locations, but also by being experts in real estate property, generally within local regions and within a specific commercial real estate sector (office, industrial or retail).

Real estate portfolio managers also deal with the life of the property. In addition to creating and approving budgets and expenses, asset managers increase value by managing the leasing of properties managed. Leasing is one of the more complex functions, and has a strong effect on portfolio value. Success in leasing is not only attained by finding and executing the highest possible leasing opportunities of each unit, but also by conducting a leasing program aligned with portfolio strategy while maintaining good relationships with existing tenants. For example, during periods of rising rent rates, long-term leases may be locked in to income-oriented properties, while for strategies focused on capital appreciation, lease terms may be shortened to take advantage of higher expected rates in the future or for property-repositioning programs.

Mentioned above, break-options have not been the subject of many studies. The reason is simple: questions center on continental Europe. In the U.K. and United States where the majority academic research (particularly real estate) is conducted and published, options are not the same and sometimes do not exist. Options to leave in favor of the tenant (an asymmetric option) are not the way Anglo-Saxon culture thinks. In that context, an option must be dual: both the tenant and the landlord have the possibility to decide what to do about the space. Some argue tenants are too protected by law in France. However, the concept of break-options similar to that known in continental Europe is slowly emerging, especially for large tenants that have bargaining power. This is why this kind of option has not been the subject of many studies. Notwithstanding, the importance of this kind of option and the ways they can be analyzed is a fundamental issue that could be the subject of much research. They have a strong influence on investors' decisions and on the structure of the market. They can be used by tenant to manage expenses and by contrast, they can be used by investors to attract tenants who are uncertain of their business plan or to negotiate higher rent. These options offer many possibilities that should be explored.

Commercial real estate investment is often analyzed in light of the length of secured cash flows, and long-term leases offer numerous advantages to investors. First, secured properties are easier to price since no assumptions have to be considered for future tenant decisions. Second, short leases raise many questions for the investor. Price if vacant, level of rent achievable, length of potential vacancy and financial incentives to concede to the tenant are questions raised only in cases of vacancy. That makes the decision more arduous. Third, strategies are more difficult to determine with short-term leases. Fourth, long-term leased properties are more liquid and can be sold more quickly. We focus on the way break-options can be considered to manage a real estate portfolio. We concentrate on the way break-options can be included in cash flows and the influence they have on portfolio management. Research of this topic is potentially large; covering it is not possible in a thesis.

We propose a new approach that considers options embedded in leases. The model can be used for valuation, portfolio management and risk management. This is the subject of the first chapter in this part. The second chapter concentrates on portfolio management when these kinds of options are considered, and questions the optimal holding period (optimal time to sell) of a real estate portfolio that accounts for break-clauses. This second article involves not only aspects arising from portfolio management, but also investigates property risk management and addresses cash-flow modeling perspectives.

The two papers presented in this part along the two chapters are:

Chapter III: Paper 1: Combining Monte Carlo Simulation and Options to Manage Risk of Real estate Portfolio

Chapter IV: Paper 2: Optimal Holding Periods of a Real Estate Portfolio according to the Leases;

Chapter III. Combining Monte Carlo Simulations and Options to Manage Risk of Real Estate Portfolio Management (2012)[1]

This study examines options embedded in leases to better account for them in the investment process, which includes acquisition, portfolio management and risk management. The third chapter of this thesis examines break clauses conceded to tenants embedded in leases contracts. We investigate inclusion of these kinds of option in a cash flow model and their effect on investment management, risk management and investment decisions. Break options are options in favor of a tenant that offer the right but not the obligation to vacate the property on a predefined date. Break options can be interpreted in various ways, but in this chapter, we analyze them exactly as financial options except we do not determine the premium of the option but explore the possibility of exercising the option. The problem with break options in real estate finance is how to determine their exercisability and how to account for them during cash flow modeling. Often, fund or asset managers consider two or three scenarios

[1] This study led to a paper that is under consideration at the Journal of Property Investment and Finance (Emerald), and we are waiting for reviews. The article was written with Michel Baroni (Essec Business School), Fabrice Barthélémy (University of Cergy-Pontoise) and Etienne Dupuy (BNP Paribas Real Estate). The article has been presented at many conferences, and has been the subject of many modifications since the first presentation. It was presented in draft state in 2010 at the American Real Estate Society's annual conference in Naples, Florida. The study garnered numerous positive comments. It was then presented in the same year at the European Real Estate Society's annual conference in Milan. The paper received many recommendations, particularly a few concerning distributions of returns. Next, we presented the paper at the American Real Estate Urban Economics Association's annual conference in 2012. The paper was refereed by Jacque Gordon from Wharton University and senior researcher for Jones Lang Lasalle in Chicago during a session chaired by Austin Jaffe from the University of State Pennsylvania. The paper received many comments and recommendations. The range of possible cash flows and returns and apprehension of break risk was appreciated. The referee recommended focusing more on risk management. Finally, the paper was presented at the French Financial Association (Association Française de Finance, AFFI) in 2011. The general topic of real estate received excellent feedback, and attendees appreciated the model. The author would like to thank participants of all these conferences for their helpful comments.

where tenants either stay or leave the property, and determine returns in two other cases.[2] They consider the possibility of a tenant leaving. Even if it accounts for multiple scenarios, this method does not consider all possibilities. During some market events, rental values rise quickly, and the tenant will not move to a more expensive space. In other cases, rental values decline dramatically, but since the rent is not high, the tenant is again not tempted to move. The problem is to determine reliably how to account for break options and determine whether an option is exercised (given rational behavior). We propose a model that addresses the break option issue and simultaneously accounts for general market risk. Our approach considers both market and idiosyncratic risk, specific risk in real estate that is a large part of leasing. Specific risk is difficult to diversify in real estate as discussed in the literature review because a large number of assets is needed. Accounting for specific risk is essential when valuing or managing a portfolio.

This chapter's method relies on combining Monte Carlo simulation with option theory. The model is comprised of a traditional discounted cash flow model that accounts for all incomes and outcomes of a real estate portfolio investment. Incomes are rent and terminal values and outcomes are expenses related to properties, if any, according to lease structures (see part I. chap. 2). On this basic model, market risk is added using Monte Carlo simulation on both prices of the portfolio and market rental values. Correlations among all simulated parameters are considered. In this way, both price and market rental value market risks are considered in cash flows. However, in real estate finance, usual income is not commensurate with market rental values. Inflows come from rent, and rent is rarely equal to market rental values (i.e., only when a lease is contracted in a rational world). In addition, units might become vacant for a period if a tenant vacates, so using option theory, we include the possibility of a change in cash flow. At the time of a break option, a let space is vacated if rent is too high in comparison to market rental values of comparable spaces. We account for such vacancies. Vacancy length is modeled using Poisson's law, a parameter that represents average vacancy length in the market. This analysis follows option theory in the sense that we analyze the possibility of a tenant exercising an option to leave exactly as a financial option. The option is exercised if at the predefined date, rent paid (passing rent) is higher than the market rental value of similar space.[3] This is combined with methods that repeat scenarios thousands of times, so the possibility of exercising an

[2] Usually referred to as a base, opportunistic or pessimistic case.

[3] This analysis is only partially true because it ignores friction and agency costs such as moving costs, broker fees etc. This point is discussed later.

option similarly repeats. Monte Carlo simulation in real estate cash flow forecasting is common, even if few papers concentrate on this point (French and Gabrielly, 2005; Baroni et al., 2007a or Hoesli et al., 2006). For each Monte Carlo scenario, we obtain a price of the portfolio, and the action is repeated thousands of times to obtain a range of possible scenarios. We use option theory to account for specific risk of a property in the portfolio, and we use Monte Carlo simulations for market risk: prices and market rental values. We thus account for both market and specific risk.

Without rationality, typical real estate cash flow models do consider options embedded in leases, and often it is only a fund manager's perception that prevails. In addition, lease structures and contracts vary widely from country to country. We demonstrate that simultaneous use of Monte Carlo simulation and option theory improves real estate portfolio management, risk management and real estate valuation. Our work contributes to extant research in several ways. First, we consider options embedded in a lease. Second, we use Monte Carlo simulation for prices and market rental values to incorporate risk in cash flow modeling. Third, we combine the two ideas to obtain a better valuation and risk management model. Our method accounts for both market and specific risk, fundamental for managing real estate portfolios because specific risk is difficult to diversify and generally persists except in very large portfolios.

Our study allows future model improvements and provides directions for use of our approach in risk management. This method is beneficial for examining difficulties investors have meeting loan payments. A model that considers options embedded in a lease has not appeared in the literature. This approach encourages investors to consider options not only as risk, but also as opportunity.

I. Introduction

Since real estate asset transactions are relatively infrequent, there is a lack of market valuations by means of which value can be estimated. Real estate portfolios are largely assessed using valuation models. Investors must add their own perceptions of risk to these models to arrive at a decision to buy or sell an asset. The most frequent financial risks they face relate to operating costs, vacancy rates, leases and liquidity. Principal opportunities for improving performance arise from operating costs, terminal values and rental income growth since leases provide mechanisms to increase payments over a lease's life. The need for appraisal in real estate business arises from the heterogeneous nature of properties; all properties differ at least in terms of location, and this is one of the most important determinants of value. A centralized Walrasian price cannot be set for trading property assets as is common for capital market securities. Absence of a market-based pricing mechanism prompts a need to improve valuation methods so they more accurately reflect risks involved in real estate.

Real estate valuation includes appraisal of the prospective price of a property. Real estate can involve nearly all of the problems encountered in the practice of valuation, though it is a more predictable asset. Following Nassim Taleb, who in his foreword to Geman (2005) emphasizes the specific nature of commodities, we emphasize some specificities of real estate assets. The first characteristic is unique location. While a security is abstract with no location and existing only as a balance sheet entry, properties possess location characteristics that make arbitrage arduous and comparison difficult. Second is temporal; a real estate asset is illiquid. Buying and selling is quite predictable in real estate, rooted in its physical nature. Buying and selling spans months in comparison to securities traded twice in seconds. Third is the size of the investment; real estate assets are large, indivisible assets. Fourth is obsolescence; a building deteriorates and does not keep its level of efficiency. Fifth is cash flow; small cash flows occur while the asset is held, and large flows occur at the time of sale. Considering all of these characteristics, the difficulties of real estate valuation are obvious.

Accepted widely by practitioners and academicians, traditional valuation methods include construction costs, comparisons with similar assets, yield capitalizations, discounted cash flows, and asset present values. These methods suffer from many limitations, particularly two inherent disadvantages: they do not consider risk and they are sensitive to parameters such as infinite growth rates of cash flows. These limitations are discussed by Fama and French (1989), Ferson and Harvey (1991), French and Gabrielli (2004) and Myers (1974), who favor the present-value approach.[4] Traditional valuation methods do not meet basic requirements such as probability distributions, standard error calculations and confidence intervals. Such limitations are problematic so we overcome these issues by suggesting a new valuation method that, using Monte Carlo simulation and options, incorporates uncertainty into the valuation process. The Monte Carlo method has long been applied to incorporate risk into simulations of many scenarios. Adding an option element to cash flows allows us to

[4] The APV approach was used in real estate finance, among others, by Hoesli et al. (2006)

consider risk borne by the lease. Simulation enables us to derive a broad range of applications and metrics such as VaR.

The first use of simulation in real estate appraisal can be found in Pyhrr (1973), who analyzes risks of real estate investments. Quantitiative methods[5] are used by Wofford (1978), Li (2000), Kelliher and Mahoney (2000), Dupuy (2003, 2004), French and Gabrielly (2004, 2005, 2006), Hoesli et al. (2006) and Baroni et al. (2007a, 2007b). The idea introduced by Pyhrr (1973) renders risk assessment explicit, and makes better use of modern financial theories and computer resources. In 1973, Pyhrr emphasized that it is possible to use quantitative methods to develop models that help investment decision-makers account for three dimensions: degree of uncertainty, time dependence and complexity. This model forms the basis for all modern models that use Monte Carlo simulation in real estate. Monte Carlo methods are computer intensive, but as early as Jaffe (1985), extensive and increasing use of the computer for real estate valuation purposes is noted. We believe a revolution is occurring in real estate risk assessment, with quantitative and probabilistic approaches employed increasingly in the computation of risk measurement and management of investment risks. Monte Carlo simulations were used extensively recently for risk management and stress testing in many financial fields. In real estate finance, Follain and Giertz (2011) develop a model that estimates the severity of low-probability events in house price movements. Following Follain and Giertz (2011), our model can be used for stress testing or risk valuation in commercial real estate.

This work builds principally on four sources: French and Gabrielli (2005), Hoesli et al. (2006), Baroni et al. (2007a) and Dupuy (2003). These papers demonstrate and present the first practical uses of Monte Carlo simulations.

French and Gabrielli (2005) propose a standardized approach, suggesting a generic forecasting software package such as Crystal Ball[6] to incorporate uncertainty into valuation (cash flows). Their article is based on the discounted cash flow model, and its authors emphasize that uncertainty comes from both lack of knowledge and imperfect information concerning inputs usually used in analysis. They particularly propose assigning a probability distribution to uncertain input variables, and are able to determine both a range of outcomes and valuations. They also argue the possibilities this approach offers such as client information on valuation uncertainty and risks involved in assessments. In conclusion, they emphasize a need to establish a standard approach.

[5] A presentation of real estate quantitative methods is reported in Jaffe and Sirmans (1994), chapter 13.

[6] Other software such as @Risk exists.

Hoesli et al. (2006) add uncertainty to the valuation process, and solve the problem of a constantly weighted average cost of capital using Monte Carlo simulations in the discount rate. They simulate a risk-free rate using Cox et al.'s (1985) dynamic model, incorporating variations into the risk-free interest rate. Moreover, the model supports the inclusion of building-specific characteristics in the risk premium, achieved by rating market states and assessing property-specific hedonic characteristics that are translated into building-specific risk premiums. The approach is similar to the scoring method used by the hedge-fund industry, and overcomes some of the most salient limitations of the discounted cash flow model. It also incorporates risk into cash flows using a time-varying weighted average cost of capital. The method does not require knowledge of an asset's value, which is normally necessary to determine the weighted average cost of capital, at least from an academic viewpoint.

Baroni et al. (2007a) propose a real estate portfolio valuation method that uses Monte Carlo simulations, simulating both rental value and price of an asset. The model is innovative by introducing uncertainty not only into cash flows, but also into asset price. Furthermore, they model vacancy rate using a uniform law, and thus incorporate the possibility of vacancy changes in the portfolio. Among other things, their approach permits elimination of one of the most profound issues in valuation; the terminal value calculation is simulated rather than being dependent on a dubiously infinite growth rate. Empirical tests using an index constructed by Baroni et al. (2005) suggest just how robust the method is in comparison to the traditional discounted cash flow approach. The approach permits measurement of risk with a given distribution of outcomes. VaR is particularly easy to compute. The paper also opens many applications for portfolio management. For example, Baroni et al. (2007b) and Barthélémy and Prigent (2009) derive optimal holding periods for a real estate portfolio, a well-known issue in finance.

Dupuy (op.cit.) introduced a combination of Monte Carlo simulations and options for real estate valuation in 2003. He considers risk borne by the real estate owner, focusing on options granted to a tenant in a lease. Dupuy concentrates on market rental values, and combines Monte Carlo simulations for these market rental values with options to demonstrate that a lease structure incorporating break clauses transforms normally distributed market rental values into a reduced set of income paths. More precisely, the author derives tenant behaviors by comparing expectations of tenant cash flows and market rental value cash flows for each simulated scenario. In addition, Dupuy derives many applications and measures regarding average time on premises and probabilities of a tenant vacating. This is not, however, incorporated in the asset's price at the end of a period.

Following Dupuy (2003), French and Gabrielli (2005) and Baroni et al. (2007a), we improve existing commercial real estate valuation methods by introducing uncertainty and risk into the valuation process. The essential contribution is that we consider the option to vacate in the lease as a financial option. We consider uncertainty in the input parameters of the model, but also uncertainty deriving from a tenant's behavior with respect to the right to vacate. Therefore, we consider both market and idiosyncratic risk. This is especially fundamental in real estate portfolios since it has long been argued that such portfolios require a large number of assets to diversify (Brown, 1988, 1991; Byrne and Lee, 2000; Callender et al., 2007). Given the difficulty of maintaining such a quantity of assets, specific risk is often not diversified well. Considering specific risks - and among them, leasing risk is one of the most important - is a key issue when managing or valuing real estate portfolios. We achieve this by combining Monte Carlo simulations and option theory.

We focus on one of the most essential determinants of the commercial lease: the lease and its structure. A lease is a rental agreement between a landlord and tenant. In Europe, lease structures vary from country to country, and are different in the U.K.[7] in comparison to the rest of continental Europe. Information concerning lease structure is an essential component of any cash-flow model because the lease provides information about expected cash flows in subsequent years. A lease usually specifies starting date, initial rent, expiry date, indexation rules and the option of the tenant to vacate the premises before expiry. These last options are called break (sometimes renewal) options (clauses). They are a particular feature of a continental European lease, and one of the greatest risks faced by European real estate investors. A break option is an asymmetric right in favor of the tenant. At the time of a break option, the tenant has the right without obligation to terminate the lease. A tenant may leave at the time of a break option, so break options are likely to cause vacancies and a hiatus of cash flows, representing the principal cash flow risk investors face. A void incurs costs and gives rise to no revenue. Premises may deteriorate and outdate more quickly, representing increased risk of capital loss. Extant rental contracts research - Miceli and Sirmans (1999) - suggests landlords attempt to minimize vacancy and turnover costs by offering discounts to long-term tenants. Landlords try to minimize the number of possible break-options conceded to tenants by offering discounts and rent-free periods.

[7] U.K. leases are usually long leases without break clauses, and only involve upward revisions of rent. However, use is changing, and options to leave (asymmetric or dual) are appearing slowly.

The proposed model considers risks underlying the lease structure in relation to market rental values, agency costs, and current rent paid in the portfolio as a whole. We use Monte Carlo simulation and apply basic option theory:

- We use Monte Carlo simulations to price a portfolio and for all market rental values of all rental spaces in a portfolio;
- We apply basic option theory to break options granted a tenant.

Each Monte Carlo scenario generates a value on every lease, achieved through three factors: simulated state of market rental value, lease structure, and tenant decisions, all according to current rent and assuming rational behavior. The model incorporates uncertainty with regard to cash flow in relation to the state of market rental values, and also incorporates it into a price component. We claim this method is more accurate and reliable than traditional methods. The method offers the possibility to compute a number of risk metrics, including a range of forward return outcomes and VaR. This makes the approach relevant to risk management purpose and reporting.

Section 2 develops the model as we focus on risks that must be considered, including vacancy, which we model with options. Section 3 describes and illustrates simulations applied to a lease. Section 4 provides applications and illustrations of the model based on a case study in which we conduct sensitivity analysis to measure the model's robustness. Section 5 summarizes conclusions.[8]

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[8] **Prerequisite:** We state here assumptions we make throughout paper and the terms we use.
We assume investors are rational and use net present value to decide on favorable investments. We assume a real estate portfolio comprised of many commercial properties. These properties are so many spaces that can be rented. The portfolio is acquired or sold as a whole during one period, and the properties in the portfolio are not acquired gradually. At the time of acquisition, the spaces comprising the portfolio can be let or not; if not, they are vacant. The space leased at acquisition may have been contracted at different dates before acquisition, and therefore have different break option and maturity dates. For simplification, we consider all leases are already contracted or may be contracted, have been signed or will be signed on January 1st. All cash flows occur on January 1st, and we do not consider quarterly payments. A 9-year lease that begins in the 1st period has its first payment during the 1st period and the last payment during the 9th period. A lease that has its first break option at year 3 may only be vacant during the 4th period, so it produces at least three cash inflows. Rent is paid one period after acquisition or the signing of a lease. Rent decided at time t is received one period after at time $t+1$.

II. The basic model

Our model is based on the non-deterministic (stochastic) approach taken as the classic discounted cash flow method.[9] Our objective is to determine, as accurately as possible, all future cash flows generated by a portfolio. Integrating risk into each cash flow enables us to compute global risk of a real estate portfolio. Our approach considers tenant behavior with respect to the lease structure of a portfolio in commercial real estate. The model estimates both risks included in cash flows generated by assets in the portfolio and risks associated with the market: prices and market rental values.

To describe these risks more precisely, we consider five sources of risk:

- indexation risk of rental value (i);
- evolution of market rental value prices (ii);
- potential vacancy rate of the portfolio (iii);
- variation of expenses over time (iv);
- market price risk (v).

Conjunction of these risk sources enables us to decompose risks related to a portfolio's cash flows. We analyze cash inflows relative to rent and then cash outflows so we can construct a valuation model for the portfolio.

Our model considers the specificity of each sub-market and each lease structure within the portfolio. We also consider n assets (or n properties) to be invested in m sub-markets. We assume $(m \leq n)$.

A. Cash Inflows

Cash flows generated by a real estate investment are comprised of rental payments. To compute future rent, three factors must be considered:

- rent evolution, or the way rent is revised in leases (i);
- evolution of market rental value for the sub-market corresponding to the asset (ii);
- evolution of possible vacancies for each lease (iii).

[9] Baroni et al. (2007a) report a clear and developed description of the traditional DCF method in comparison to a stochastic model.

For $\forall i = 1,\ldots,n,\ S_i$ is the subset of dates in which the option to leave can be exercised for space *i*. We analyze rent for a space *i* belonging to a sub-market $j \in [1, m]$. Its rent at time t $(t = 0,\cdots,T)$ is $Rent_{t,i}$, its corresponding market rental value is $MRV_{t,i}^{j}$, and the index (when applicable) mentioned in the lease for rent revision is $I_{t,j}^{Rent_{t,i}}$.

The space may be let or not let. If let, the tenant has or does not have the possibility of leaving at a predetermined date before the term of the lease (i.e., the break option). When the lease terminates, both tenant and landlord can decide to continue with the lease or not, with some limitations we consider irrelevant here. The end of the lease also represents a break option but is symmetric since both tenant and landlord can exercise it, even if often at varying costs. Thus, uncertainty regarding changes in rent over time arise primarily from possibilities that the break option will be exercised,[10] and the length of any vacancy periods. This is precisely what we investigate in our model. We examine risk presented by break options and the length of vacancy in terms of cash flows and portfolio valuation. At a break option point, a tenant has two possible choices: staying or leaving. From the owner's perspective, this is translated into continuity of cash flows versus a void period, or even negative cash flow. Given an assumption of rational behaviors, the model determines choices open to a tenant, incorporating consequences in cash flows. Faced with a break option, both tenant and landlord wish to increase wealth. The landlord wants to hedge revenues and increase building value. The tenant wants to minimize both current rent paid and expenses related to the premises. Assuming rational behavior, when faced with a break option, the tenant stays on the premises only if rent is not too high in comparison to current market rental values available for similar spaces on the market, plus relevant transaction or agency costs.[11] We model vacancy length using a random discrete

[10] We do not consider bankruptcy even though the risk is relevant.

[11] There is an analogy in classic financial derivatives. The break option works as a valuable European option. In finance, a European option is a contract between a buyer and a seller that gives the buyer the right without obligation to buy or sell an underlying asset at a later predefined date at an agreed price. In return for the option, the seller collects a premium from the buyer. In a capital market, a rational player exercises a European option at maturity as soon as it terminates if the sum of money (i.e., its underlying price) is above a predefined point. By analogy, at the time of a break option, the tenant has the right without obligation to terminate the lease. A rational tenant terminates the lease only if its option to leave—its break option—is in the money (i.e., if the market rental value is below rent). In contrast to traditional financial analysis, we do not value the option's premium. In our model, the tenant is tempted to leave as soon as the market rental value for available identical space is much less than current rent.

Note: A call option is in the money if its underlying price is above a predefined price: $S_t > K$. A rational player exercises the call to buy the underlying asset at a cheaper price. A call option is at the money if the price of the distribution: Poisson's law. In this way, the prospects of a vacant space being let or remaining vacant are considered.

At each period, there exist three outcomes of modeled $Rent_{t,i}$:

- If a lease has just been contracted, at period *t*, the lease is around market price.

$$Rent_{t,i} = MRV_{t,i}^{j} + \varepsilon_i,\ \forall t \geq 0 \text{, with } \mathrm{E}\left(\varepsilon_i\right) = 0 \text{ and } \mathrm{V}\left(\varepsilon_i\right) = \sigma_i^2$$

Noise ε_i arises from market incompleteness and inefficiencies (e.g., information regarding real estate transactions is never fully available.

- For current lease, the rent at period *t* is:

$$Rent_{t,i} = f(Rent_{t-1,i}, \Delta I_{t,j}^{Rent_{t,i}}), \forall t \geq 0,\ \forall i,\ \forall j,$$

$I_{t,j}^{Rent_{t,i}}$ represents the index to which the rent has been fixed from the beginning of the lease. This case remains valid even for the first period of the lease ($t = 0$). In this case, the lease has already been contracted at the time the portfolio is purchased. The beginning of analysis does not usually correspond to beginning of leases in the portfolio since the tenant may be in place before acquisition.

- At period *t*, the premises may be vacant. In this case, no rent is paid:

$$Rent_{t,i} = 0, \forall t, \forall i$$

Hence, the model for rent at period *t* of premises *i* may be summarized as:

$$\forall t,\ \forall i,\ \forall j,\ Rent_{t,i} = \begin{cases} MRV_{t,i}^{j} + \varepsilon_i, \text{ with } \mathrm{E}\left(\varepsilon_i\right) = 0 \text{ and } \mathrm{V}\left(\varepsilon_i\right) = \sigma_i^2 & \text{(a)} \\ f(Rent_{t-1,i}, \Delta I_{t,j}^{Rent_{t,i}}) & \text{(b)} \\ 0 & \text{(c)} \end{cases}$$

The essential characteristic of each space is to be let or vacant at period *t*. The model focuses on dynamics between two consecutive periods; for example, period *t* and period *t+1*:

- Does a let space remain let or not?
- Does a vacant space remain vacant or not?

Determining the way a current or potential tenant will behave is difficult. In our context, it means determining if a vacant space attracts a tenant and if a let space

underlying asset it is written on equals the strike price: $S_t = K$. A call option is out of the money if its underlying price is below a predefined price: $S_t < K$.

keeps its tenant on the premises. The purpose is to find a rule or principle that can be used as a standard for two predefined cases, (i) and (ii), which can be used rationally in the context of real estate portfolio management and valuation. We propose two rules, one to decide if a let space remains let when facing a break option (i) and the other to determine how long a vacant space remains vacant (ii).

- (i) The first rule is based on a comparison between current rent and market rental value available for similar space in the same market. Under the assumption of rational behavior, the option to leave is exercised by the tenant only if it is the time of a break option $\left(t \in S_t\right)$ and if rent currently paid is too high in comparison to current market value.[12] Mathematically, this can be written as:

  $\forall i = 1,\dots,n,\ \forall i = 1,\dots,m,\ \forall t \in S_i$, if

  $\dfrac{Rent_{t,i}}{MRV_{t,i}^{j}} \geq 1+\alpha$, then $Rent_{t+1,i} = 0$, for $\alpha > 0$.

  If there is no possible break option at period t ($t \notin S_i$), the space remains let by contract at period t+1. It also remains let when there is a possible option to leave ($t \in S_i$) that goes unexercised.

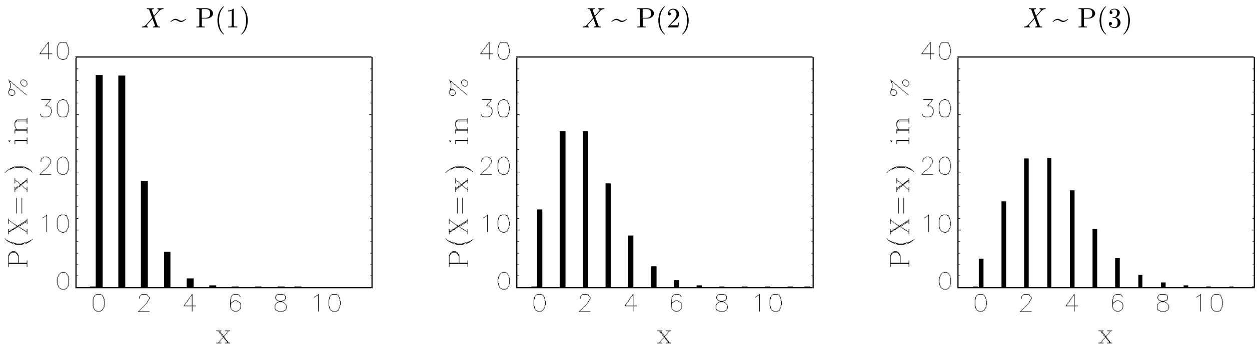
- (ii) The second issue faced by the landlord is length of vacancy. Once a vacant space exists in a portfolio, it is difficult to rationally infer the length of time the space remains vacant. Length of vacancy may be months or years, varying widely from one space to another depending on the state of the sub-market, location, and state and quality of the premises. To model these possibilities in vacancy length, we define a random variable *X* as the number of periods during which the premises remain vacant. Poisson's law is used to model vacancy length:

  $X \sim P(X = k) = \dfrac{\lambda^k e^{-\lambda}}{k!}$

  where *k* is the length of vacancy and *λ* is a positive real number equal to the expected number of occurrences during an interval. In our case, the expected number of occurrences equals average

[12] In practice, various events can occur. Landlords may concede a revision of the rental level to prevent a void situation; at the same time, tenants may prefer to stay on the premises to save transaction costs. Negotiations are common.

vacancy length in the sub-market. We illustrate Poisson's law in Graph III-1.



Graph III-1 - Illustrations of Poisson's law for average vacancy lengths of 1, 2 and 3 periods

This representation determines vacancy length - and its underlying associated probability - independently of the last vacancy period as soon as a known average vacancy length (λ) for each sub-market is available. This way, length of vacancy is modeled as a random variable that can take a different value for each simulation, instead of a chance, fixed value.

The dynamic between two consecutives periods is summarized in Table III-1.

t \ t+1	Let	Vacant
Let	No break option $\begin{cases} \text{Break option} \\ \dfrac{Rent_{t,i}}{MRV_{t,i}} \leq 1+\alpha \end{cases}$	$\begin{cases} \text{Break option} \\ \dfrac{Rent_{t,i}}{MRV_{t,i}} \geq 1+\alpha \end{cases}$
Vacant	Let (a tenant was found)	Vacant

Table III-1 – Dynamics between two consecutive periods

B. Cash outflows

Cash outflows represent various expenses related to a building arising for the landlord, and they are threefold:

- Current expenses link to regular expenses such as insurance, repairs and maintenance. Let $Exp^*_{t,i}$ be the expected current expenses during period t for space i and $\nu_{t,i}$ the probability of realizing those expenses. Expected current expenses at period t for premises i can be expressed as:
  $\forall i = 1,\ldots,n, \forall t,\ Exp_{t,i} = \nu_{t,i} \times Exp^*_{t,i}$
- Capital expenses are expenses related to maintenance or refurbishment, which have direct influence on the value of the asset or its marketability (e.g., roof repairs, ventilation or elevator replacement). Let $Wk_{t,i}$ be the expected potential work (capital expenditure) at period t for premises i and $\kappa_{t,i}$ the probability of such work being necessary. Expected capital expenses during period t can be formulated as:
  $\forall i = 1,\ldots,n, \forall t,\ Wk_{t,i} = \kappa_{t,i} \times Wk^*_{t,i}$
- Vacancy charges correspond to expenses arising only during vacancy such as land or local taxes, heating, cooling, security or marketing expenses (e.g., billboards). Let $Cv^*_{t,i}$ be expected vacancy expenses during t for premises i and $v_{t,i}$ the probability that such expenses will arise. Expected vacancy charges during period t can be formulated as:
  $\forall i = 1,\ldots,n, \forall t,\ Cv_{t,i} = v_{t,i} \times Cv^*_{t,i}$

C. Free cash flows

We are now able to determine the free cash flows we use to compute the present value of a real estate portfolio. These cash flows incorporate simulations of rent and the behavioral model of tenant choice. The free cash flows generated by the portfolio are comprised principally of cash inflows from rent and tax reductions due to depreciation, cash outflows associated with expenses, a terminal value and capital gains tax, if any. The terminal value - and capital gains tax - arises only when the portfolio is sold at the end of the holding period. Through assumption and for simplification, we treat the entire portfolio as sold at the end of the period ($t = T$).

- Free cash flows of space i at period t from period t ($t \in [1, T-1]$), where the portfolio is sold at period T, are expressed as:
  $\forall i = 1, \ldots, n, \; \forall t = 1, \; \ldots, T-1,$
  $$FCF_{t,i} = (1-\tau)\left(Rent_{t,i} - Exp_{t,i} - Wk_{t,i} - Cv_{t,i}\right) + \tau Dep_{t,i}$$
  where $Dep_{t,i}$ is the element of depreciation at time t corresponding to space i, and τ is the tax rate.
- Free cash flows for period $t = 1, \ldots, T-1$, of a portfolio comprised of n spaces to let is evaluated as:
  $\forall i = 1, \ldots n, \forall t = 1, \ldots, T-1,$
  $$FCF_t = (1-\tau)\sum_{i=1}^{n}\left(Rent_{t,i} - Exp_{t,i} - Wk_{t,i} - Cv_{t,i}\right) + \tau\sum_{i=1}^{n}\left(Dep_{t,i}\right)$$
- Free cash flows at time T at the end of the holding period is:
  $\forall i = 1, \ldots, n,$
  $$FCF_T = (1-\tau)\sum_{i=1}^{n}\left(Rent_{T,i} - Exp_{T,i} - Wk_{T,i} - Cv_{T,i}\right) + \tau\sum_{i=1}^{n}\left(Dep_{T,i}\right) + P_T - \tau \times PV$$
  where P_T is the expected portfolio terminal value at the end of the holding period and PV the capital gain (plus value) at the end of the investment.

D. Discount Rate Choice

The choice that must be made between cost of equity and cost of capital when discounting cash flows is an issue in itself, and extends beyond this thesis. Since we do not include debt cash flows in our model, a classic weighted average cost of capital is used, and the investor is assumed to be in a situation where this is relevant. We denote k as the weighted average cost of capital, used as the discount rate.

III. Applying a Monte Carlo simulation: simulating prices, market rental values and rent

To elaborate the on model, terminal value, market rental values and rent, including indexation, must be simulated.

A. Terminal value

Following Baroni et al. (2007a), our model simulates the price of a portfolio. Terminal value impacts significantly portfolio valuation, and incorporation of uncertainty in the terminal value is one of the major strengths of the model.[13] We suppose the following geometric Brownian motion governs asset prices.

$$\frac{dP_t}{P_t} = \mu_P dt + \sigma_P W_t^P$$

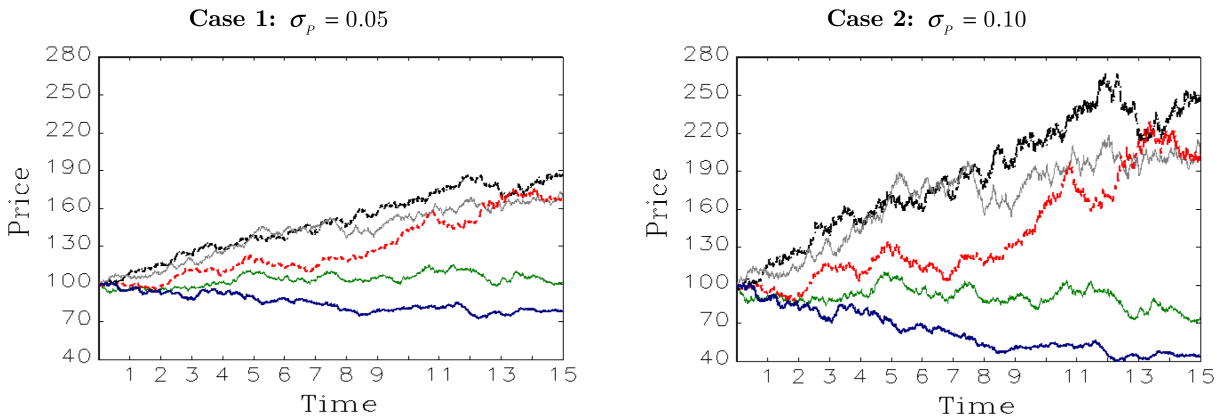
This equation assumes commercial real estate returns can be modeled as a simple diffusion process, where parameters μ_P and σ_P correspond to trend and volatility. As an example, we present paths of the price for $P_0 = 0$, and of the histogram of possible prices, with various values for μ_P and σ_P.

As an illustration, Graph III-2 uses 5 simulated paths to show two price dynamics with different volatilities, the trend being the same:

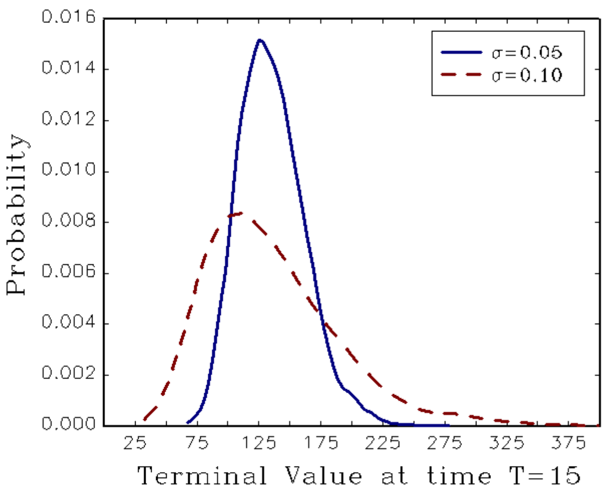
- Case 1: $\frac{dP_t}{P_t} = 0.02\ dt + 0.05\ dW_t^P$
- Case 2: $\frac{dP_t}{P_t} = 0.02\ dt + 0.10\ dW_t^P$

For each case, an estimated density function of random variable P_t is represented for t=15 and 5,000 paths in Graph III-3.

[13] Terminal values are sensitive to the leasing status of properties in the portfolio. The value of a vacant property suffers from its leasing status (liquidity and attractiveness). To clarify and simplify the presentation here, we do not considier vacancies in terminal values. Vacancy is nonetheless considered in current cash flows. This issue could be solved with an indicative function (1_{vacant}) that affects the value of vacant properties.



Graph III-2 - Five simulated price paths



Graph III-3 - Estimated density function for prices at time t = 15

B. Market rental values (MRV)

We consider the sub-market of each asset since sub-market specifications are fundamental. The dictum *location, location, location* is not tired as Bourassa et al. (1999, 2003) argues, but is a necessary condition for accurate estimates of portfolios. Following the authors mentioned previously, we consider sub-markets to model market rental values indices; we use an index per sub-market.

Each space i that is part of the portfolio generates a rent $Rent_{t,i}$ (rent can be 0) during period t. Each space i also has a market rental value at time t, denoted $MRV_{t,i}^{j}$. This market rental value follows a dynamic based on the index of its sub-market. Two spaces located in the same sub-market have the same market rental value dynamic, though divergences in values occur due to differences in size, floor level or any similar specification. We assume two spaces in the same sub-market with varying characteristics, following the same dynamics but possibly with different values. We define $I_{t,j}^{MRV^j}$ as the index of the market rental value in sub-market j during period t. We assume in the same way for price that the following geometric Brownian motion governs all indices of market rental values:

$$\forall j = 1,\dots,m, \qquad \frac{dI_{t,j}^{MRV^j}}{I_{t,j}^{MRV^j}} = \mu_{I_{t,j}^{MRV^j}} dt + \sigma_{I_{t,j}^{MRV^j}} dW_t^{I_{t,j}^{MRV^j}}$$

Indices representing development of market rental value in all sub-markets are defined by their trends and volatilities. Various market rental values can be simulated to represent a diversified portfolio invested in various sub-markets. To simulate them, estimation of all parameters $\mu_{I_{t,j}^{MRV}}$ and $\sigma_{I_{t,j}^{MRV}}$ for all sub-markets j is necessary. The market rental value of space i in sub-market j at time t is:

$$\forall i = 1,\dots,n,\ \forall j = 1,\dots,m,\ \forall t,\ MRV_{t,i}^{j} = f(MRV_{t-1,i}^{j}, \Delta I_{t,j}^{MRV_j})$$

The last equation shows the need to start with initial market rental value $MRV_{0,i}$, $\forall i = 1,...,n$. This is why the initial market rental value must be estimated.[14]

Given the dependence (i.e., correlation) between the price of a real estate portfolio and market rental values of which this portfolio is comprised, it is not possible to simulate processes independently. It is necessary not only to consider correlations between indices of market rental values and price, but between market rental value indices themselves. Simulation requires estimation of correlations between price and all market rental value indices, and between indices themselves. It is therefore necessary to estimate all $\rho_{P/I_{t,j}^{MRV}}$ and all $\rho_{I_{t,j}^{MRV}/I_{t,j'}^{MRV}}$:

$$\forall j = 1,\dots,m,\ \rho_{P/I_{t,j}^{MRV_j}} = dW_t^P dW_t^{I_{t,j}^{MRV_j}}$$

$$\forall j = 1,\dots,m,\ \forall j' = 1,\dots,m,\quad j \neq j'\ \rho_{I_{t,j}^{MRV_j}/I_{t,j'}^{MRV_j}} = dW_t^{I_{t,j}^{MRV_j}}\ dW_t^{I_{t,j'}^{MRV_j}}$$

Graph III-13 shows an illustration.

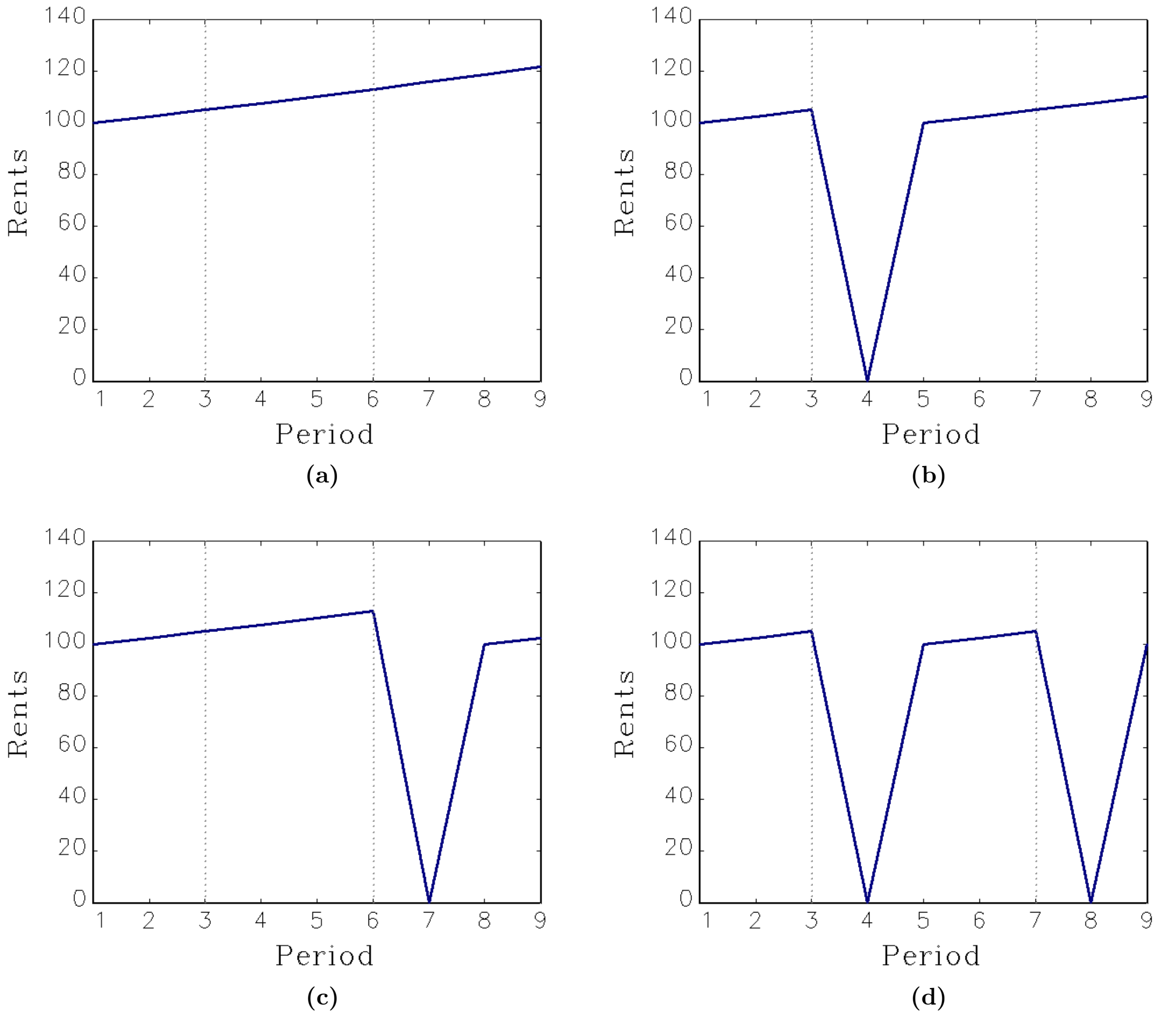
[14] Market rental values are available freely for nearly all markets (i.e., on brokers' websites).

C. Simulation of the rent

As detailed in section 2, the lease structure must be considered when simulating rent. For each lease and at break dates ($t \in S_i, \forall i = 1, \ldots, n,$), the market rental value of each space is compared to current rent, and the model demonstrates whether the option to leave should be exercised.

For illustrative purposes, we present in Graph III-4 all possible cases that occur for a classic French commercial lease in the case where vacancy length is fixed for one year. This lease takes the form of a 9-year contract with two break options: to leave the premises before the end of the contract at the end of years 3 and 6. Consider a periodic indexation set to 2.5%. The market rental value is fixed at 100 during the lease. The lease begins at period $t = 1$, and four cases occur in Graph III-4:

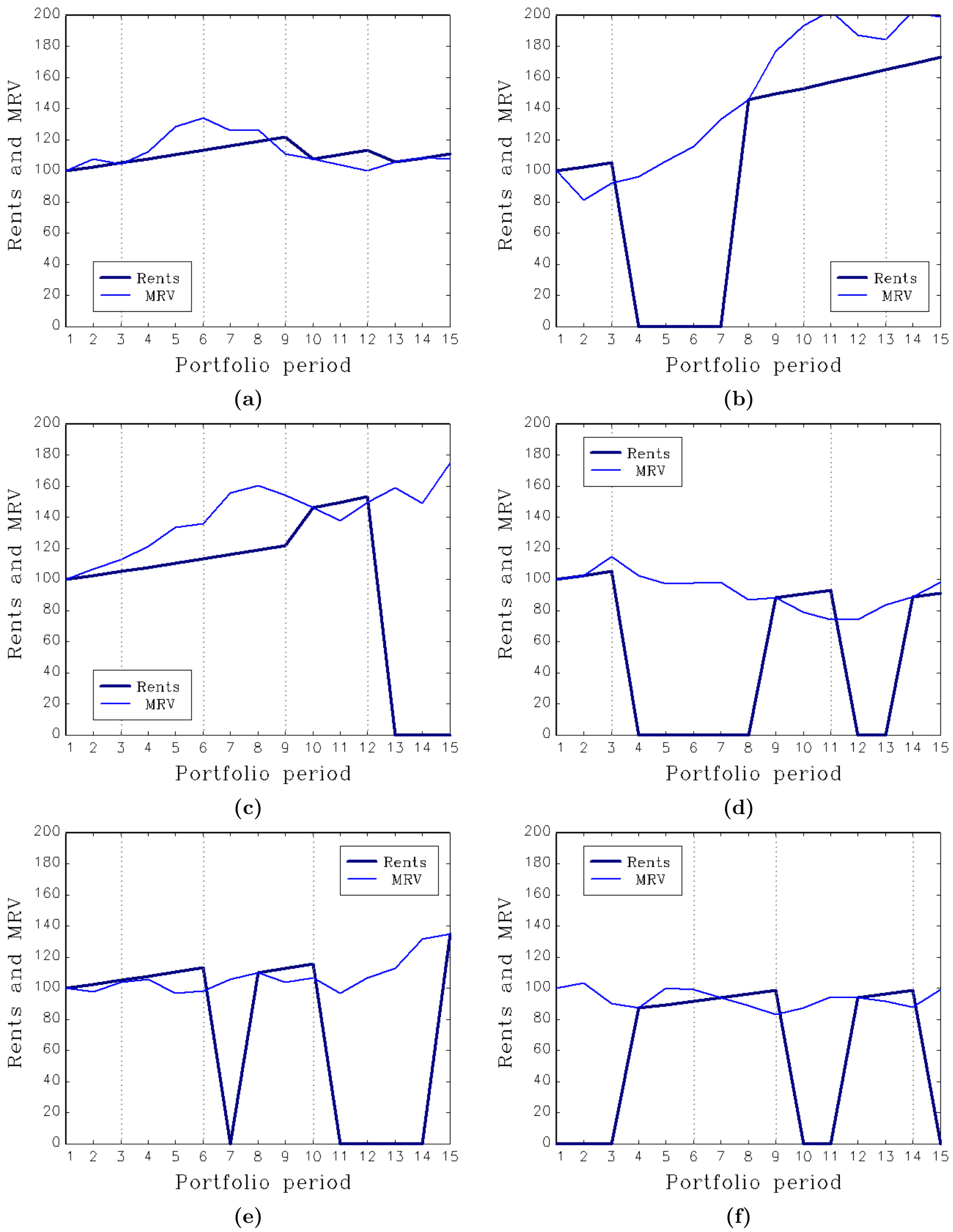
a) The space remains let during the lease and no break options are exercised;
b) The first break option (end of period 3) is exercised. After one year of vacancy, a new tenant enters the premises and stays until the end of the new lease;
c) The second break option (end of 6th period) is exercised. A new tenant enters the space after one year of vacancy;
d) Each current tenant exercises the first break option each time the option arises, and the landlord faces two years of vacancy during the 4th and 8th periods.



Graph III-4 - Four possible cases of a French lease

In all cases, rent returns to the market rental value after each break option is exercised. Consequently, a break option represents opportunity for the tenant or risk for the landlord.

D. Simulation of rent and market rental value



Graph III-5[15] - Six illustrations of our method used in a French lease

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[15] Comments to Graph III-5:

The cases represented in Graph III-5 form a panel for all cases generated through simulation.

We introduce market rental value and rent received by considering the dynamics represented by vacancy length. Graph III-5 presents rent resulting from six simulated cases for a French lease (9-year lease with break options at years 3 and 6) over 15 years. The bold line represents rent paid and the lighter line represents market rental value. In these graphs, rent is indexed annually at 2.5%, and the average vacancy length is 2 years ($\lambda = 2$). We assume a tenant decides to move as soon as rent is higher than the market rental value at the date of a break option ($\alpha = 0$). The market rental value index follows $N(0.02; 0.1)$, and Graph III-5(f) is a case in which no rent is paid during initial periods.

The Monte Carlo method creates a large number of scenarios, and each presents rent paid by one space during a simulation. By repeating this hundreds of times, we calculate the average of all simulations and obtain rent expected during the period. Graph III-6 represents three paths for rent, market rental value and their combinations for a French lease. Graph III-7 represents the averages of rent and market rental values for the lease (3/6/9). As before, market rental values are represented by $N(0.02; 0.10)$ and rent is indexed at 2.5% per annum. The average vacancy length is 2 years ($\lambda = 2$) and we assume $\alpha = 0$. We obtain the expectation of the Brownian market rental value (determined analytically) and expected rent. As predicted, rent is below market rental value.

(a) - None of the break options are exercised by the first tenant during the course of its lease until period 9. However, the space is vacated when the lease terminates, but the landlord does not face vacancy because a new tenant enters the space immediately, so there is no void period. After three years on the premises, the second tenant leaves the space and, once more, a tenant immediately enters the premises without void.

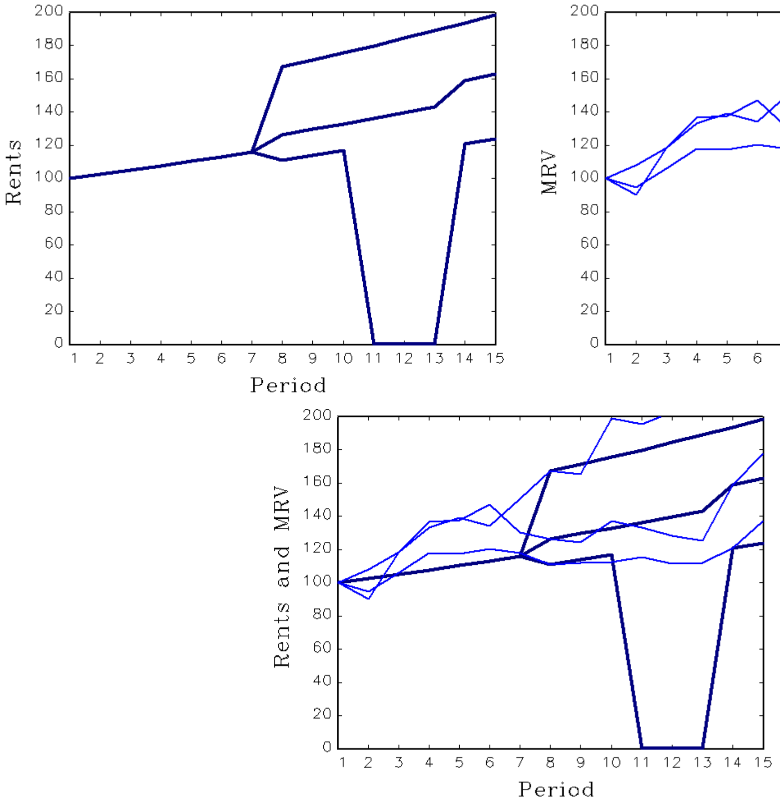
(b) - At the end of year 3, the tenant finds a cheaper place elsewhere, and the space is vacated. The vacancy lasts 4 years until the 8th period when a new tenant is found.

(c) - The first tenant stays on the property until the end of the contract, leaving after nine years. At that time, a new tenant enters the space for higher rent and leaves the property after three years since the market rental value has hence fallen. The space remains vacant until the end of the simulation.

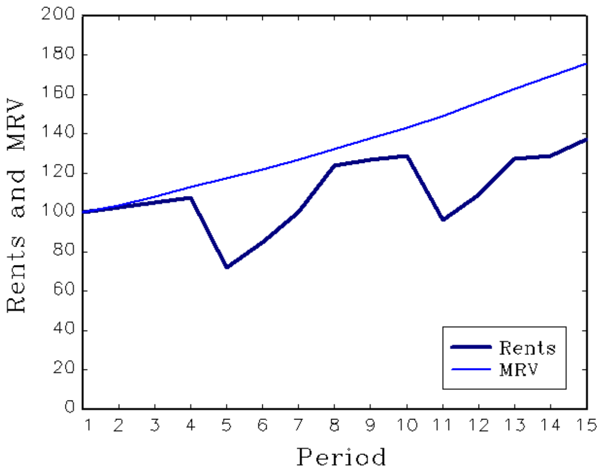
(d) - The first tenant leaves the property at its first break option, and the space remains vacant for 5 years. After 3 years on the premises, the second tenant decides to leave and the space has two years of void before a third tenant enters the property.

(e) - The first tenant stays 6 years. After one year of vacancy, a second tenant lets the space. Three years later, the second tenant leaves and the space is vacant for 4 years.

(f) - The property is vacant at the beginning of the simulation, and remains vacant for three years. The first tenant enters the property at period 4 and stays six years. The space remains vacant for two years and then a second tenant lets the space and leaves after three years.



Graph III-6 - Three paths for market rental value and for rent in a French lease.



Graph III-7 - Average of 10,000 scenarios for rent and market rental values in a French lease

E. Distribution of rent

To illustrate the relevance of the model, we present a histogram of rent paid in a French lease for each period. The lease is assumed to have been signed initially for 9 years with 2 break options at years 3 and 6. When broken and after the vacancy period, the new lease is contracted under the same terms (3/6/9). Graph III-14 in Appendix 2 shows the change from the lease structure, and highlights the importance of taking options offered to a tenant.

F. Simulation of free cash flows

In this section, simulations both of rent and terminal value of the portfolio are combined. The classic discounted cash flow model is used to price net present value of the real estate portfolio. All future free cash flows are discounted at discount rate *k*. This enables us to attach a value to the portfolio. The distribution obtained from all scenarios can be used for risk management.

$$P_0 = \sum_{\theta=1}^{T-1} \frac{FCF_t}{(1+k)^{\theta}} + \frac{FCF_T}{(1+k)^T}$$

or more precisely:

$$P_0 = \sum_{\theta=1}^{T} \frac{(1-\tau)\sum_{i=1}^{n}\left(Rent_{t,i} - Exp_{t,i} - Wk_{t,i} - Cv_{t,i}\right) + \tau\sum_{i=1}^{n}\left(Dep_{t,i}\right)}{(1+k)^{\theta}} + \frac{P_T - \tau \times PV}{(1+k)^T}$$

Applying the Monte Carlo method, the process is repeated hundreds of times, and we obtain a large number of prices for the portfolio. The average estimates the portfolio's value.

The parameters that must be estimated to implement the model include:

- trend and volatility of price: μ_P and σ_P;
- trends and volatilities of all market rental value indices $\mu_{I_{t,j}^{MRV}}$ and $\sigma_{I_{t,j}^{MRV}}$, $\forall j = 1,\ldots,m$;
- correlations between all simulated processes.

The lease structures of each space, initial rent and indexations for each lease are known in the contract. The initial market rental values, initial market price and tax rate are observable or can be estimated. Discount rate k corresponds to the weighted average cost of capital, and the depreciation rate and its impact on cash flows are country-specific and observable. For simplification, expenditure growth is constant but if necessary and if sufficient information is available, it could be simulated. The next section deals with valuation of a portfolio and provides detail on free cash flow simulations.

IV. Application of the model

A. Valuation of a portfolio

Using the framework, we simulate both cash flows generated by leases and the terminal value of a commercial real estate portfolio. The method is based on the modeling of rent, expenditures and price dynamics. We provide a model that investors can use to analyze portfolio risk associated with lease structures. We also test robustness of the model using a sample.

The model preserves a degree of freedom for all variables and possible relationships/correlations. Modeling allows incorporation of experiences from real estate portfolio managers and local agencies. For example, an asset manager might possess empirical knowledge of a portfolio's ability to maintain tenants or may be an especially skilled negotiator, and use these in a simulation.[16] The greater the experience, the better the model should prove in simulations in the sense that the dynamics represented in the model should be closer to reality.

We take one example to compute the value of annual inflows from a portfolio over a 15-year period. We show in Graph III-16 the histogram of possible rent produced by the portfolio each year. We focus particularly on parameters that have the greatest impact on both a tenant's behavior and valuation of the portfolio: vacancy length (λ) and the difference between market rental values and rent whose level prompts a tenant to move (α).

The commercial real estate portfolio we consider is a real portfolio comprised of 16 lettable spaces. The portfolio holds 6 properties, acquired during construction

[16] An experienced asset manager can increase or decrease the decision factor to criteria.

between 2005 and 2009 (the last completion of a building). The portfolio was sold in 2010 for 100 M€. All spaces are located in the suburbs of Paris, France, driven by the same sub-market index. Therefore, $j = 1$ (the number of sub-markets for market rental value index $I_{t,j}^{MRV^j}$). The sub-market index follows normal law $N(0.04; 0.08)$ and market rental values of the spaces follow $N(0.04; 0.08)$.

$$\forall t,\ I_{t,1}^{MRV_t^1} \sim N(0.04; 0.08)$$

where j is unique in our case (same sub-market). Regarding annual rent review, we consider a fixed increase of 2.5% per annum. In practice, differences often occur with maximum indexation: a cap, fixed indexation, or upward revision only.

$$\forall t,\ \forall i,\ I_{t,1}^{Rent_{t,i}} = 2.5\%$$

Passing rent and rental values of all premises at the time of acquisition are displayed in Table III-2. One space identified with lease 14 is initially vacant, some are under-let (initial rent is below market price) as is the cases for leases 8, 9, 10, 11. Some are over-let (initial rent is above market price) as is the cases for leases 1, 2, 4, 5, 13, 16. The remaining portfolio is let at market price: leases 3, 6, 7, 12, 15. Simply and without simulation, we predict which tenants are more difficult to maintain. Holders of over-let leases are more tempted to move to cheaper places.

(in €)	Market rental values of the spaces at acquisition	Passing rents at acquisition
Lease 1	1 000 000	1 100 000
Lease 2	500 000	550 000
Lease 3	300 000	300 000
Lease 4	500 000	550 000
Lease 5	600 000	620 000
Lease 6	150 000	150 000
Lease 7	600 000	600 000
Lease 8	700 000	650 000
Lease 9	600 000	550 000
Lease 10	500 000	450 000
Lease 11	600 000	550 000
Lease 12	300 000	300 000
Lease 13	300 000	330 000
Lease 14	150 000	0
Lease 15	500 000	500 000
Lease 16	200 000	220 000
TOTAL	**7 500 000**	**7 420 000**

Table III-2 - Passing rent produced and rental values of the 16 premises

The portfolio is purchased entirely in one period, so the buyer acquires a portfolio that exists and is under management. Properties have been marketed and let for various periods, and the leases are running with various maturity dates and lease structures. The current lease structure is shown in Table III-3.

period of acquisition	**First period of break option**	**Second period of break option**	**Third period of break option**	**End of lease**
Lease 1	6			**9**
Lease 2	3			**6**
Lease 3	4			**7**
Lease 4				**3**
Lease 5	1			**4**
Lease 6	2	5	8	**11**
Lease 7	6			
Lease 8	1			**7**
Lease 9				**2**
Lease 10				**8**
Lease 11	1			**4**
Lease 12				**5**
Lease 13	6			**9**
Lease 14				
Lease 15	5			**8**
Lease 16	2	5	8	**11**

Table III-3 - Current lease structure of the portfolio

Among leases in the portfolio, some have just been contracted, some terminate within a short period and some are longer-term and include numerous break options. All leases began prior to the purchase date of the portfolio. The seller waited to hold a secure portfolio (nearly fully let) to market it. Period 1 corresponds to the period between date 0 ($t = 0$) and 1 ($t = 1$). A tenant who has a break option in year 1 will pay rent once and can then break the lease at the end of the first year. This is illustrated in the Graph III-15 in Appendix 3.

If a space becomes vacant, a new lease must be drawn up. This can be done under a predefined lease structure after a period of vacancy, or immediately after the lease terminates. Lease structures depend on size and configuration of the premises to let and on targeted tenants. The lease structures applicable to the successor tenant are shown in Table III-4. For example, if lease 10 is broken, the subsequent tenant enters the property under a traditional French leasing contract (3/6/9).

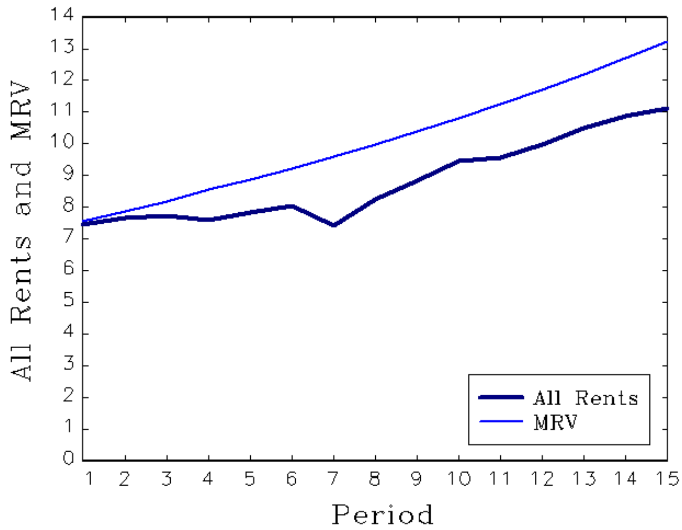
In case of a new contract	**First period of break option**	**Second period of break option**	**Third period of break option**	**End of lease**
Lease 1	6			**9**
Lease 2	6			**9**
Lease 3	3	6		**9**
Lease 4	6			**9**
Lease 5				**9**
Lease 6	3	6	9	**12**
Lease 7	6			**9**
Lease 8	6			**9**
Lease 9	6			**9**
Lease 10	3	6		**9**
Lease 11	6			**9**
Lease 12				**9**
Lease 13	6			**9**
Lease 14	3	6		**9**
Lease 15	6			**9**
Lease 16	3	6	9	**12**

Table III-4 - Lease structures of all premises in the portfolio for cases of re-letting

The initial price of the portfolio is 100 M€, and it follows a Geometric Brownian Motion driven by Normal law $N(0.02; 0.1)$. The correlation between price and rental sub-market index is 60%.[17] The entire portfolio is invested in the same sub-market, a district in Paris' suburbs. Void length follows Poisson's law of parameters 2 (average vacancy length is 2 years). Discount rate k is constant during the simulation, fixed at 6.5%. We assume no arbitrage is possible during the holding period. As stated above, all cash flows are annual and occur on the same date. The investor is a transparent fund not subject to taxes ($\tau = 0$) (Baroni et al. 2007a; Hoesli et al. 2006). For simplification and to clarify results, we do not consider expenses (e.g., current expenditure, capital expenditure and vacancy charges). In Graph III-16 in Appendix 4, we show the distribution of rent during the 15 years of the simulation.

In Graph III-8, we show average rent and market rental values produced by the portfolio throughout the simulations.

[17] Estimated from Property Market Analysis database from 1981 to 2010 for Paris' suburb sub-market.



Graph III-8 - Average rent and market rental value produced by the portfolio

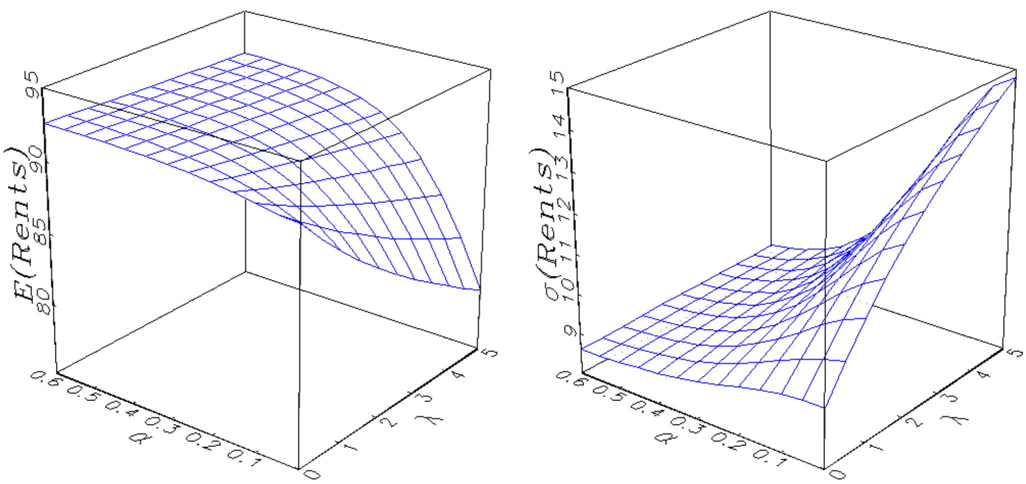
The model also displays average rent paid for each lease, so risky leases can be forecasted and analyzed, which assist decision-making.

B. Sensitivity analysis and model limitations

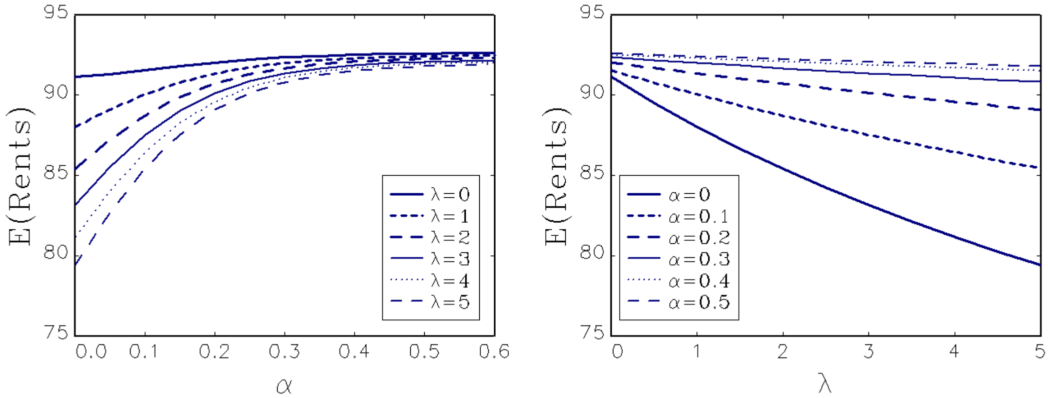
We analyze sensitivity of the model to length of vacancy (λ) and criteria for the decision to move, the moving criteria decision (α). We highlight the robustness of the model, but note some of its limitations. Sensitivity analysis is conducted using the previous portfolio under the same initial conditions; the only changes are λ and α.

Both parameters impact vacancies (on vacancy probability or length). The criterion for the decision to move increases or decreases the number of break options exercised and therefore must have a large impact on vacancies. Average length of vacancy determines how long a space remains vacant and therefore how long it generates costs instead of revenues. Graphs represent the sum of expected rent (Graph III-9, Graph III-10 and Graph III-11), portfolio value (Graph III-12) and associated standard errors.

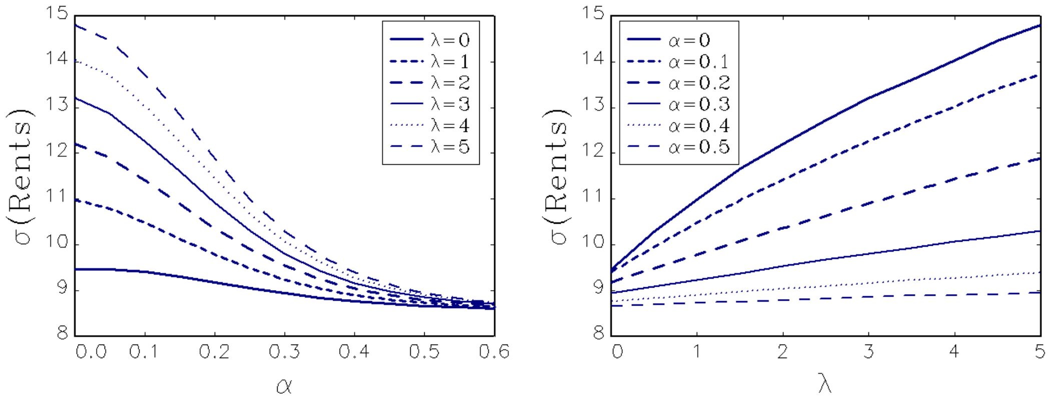
As predicted, expected rent decreases when length of vacancy increases. At the same time, rent decreases when the moving criteria decision applies more strictly (α = 0, for example). It is interesting to note that rent produced by the portfolio is affected more strongly by length of vacancy over time than by moving criteria. This is due to the lease structure. Once a tenant signs a new lease at rent close to the market rental value, predictable income is usually secured for some years, and the impact on cash flow is far less. Risk associated with these two parameters is represented by standard errors. The graph demonstrates that these two parameters have a large influence on the risk of portfolio value and also on risk associated with resulting cash flows. This result is consistent with investor fear concerning risk of vacancy. They usually understand this vacancy consists of costs it generates and the time it takes to re-let. Results strengthen evidence that vacancy is the most important factor influencing the value of a classic asset (for high-yield assets, vacancy has a higher impact with respect to cash flow volatility than low-yield assets). At the same time, volatility of the cash flow is barely influenced by length of vacancy when costs of moving are high. This highlights the interest investors have in making improvements chargeable to tenants so enhanced future cash flows are secured. According to this framework, landlords should pay for maintenance or refurbishment in exchange for surrender of break options.



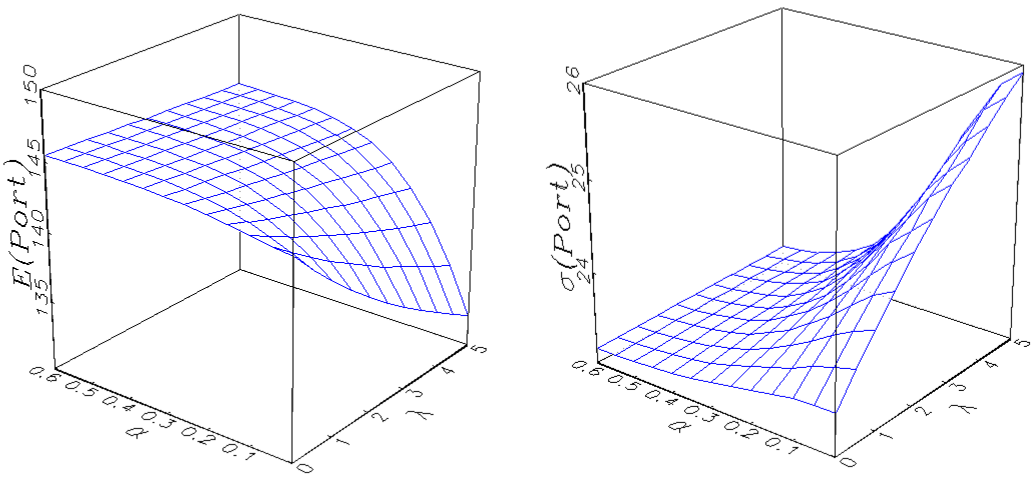
Graph III-9 - Expectation and standard error of rent in the portfolio as a function of the moving criterion decision (α) and expected length of vacancy (λ).



Graph III-10 - Expectation of rent in the portfolio as a function of either the moving criterion decision (α) or expected length of vacancy (λ).



Graph III-11 - Standard error of rent in the portfolio as a function of either the moving criterion decision (α) or expected length of vacancy (λ).



Graph III-12 - Expectation and standard error of price of the portfolio as a function of the moving criterion decision (α) and expected length of vacancy (λ).

One of the most obvious limitations of the model is the assumption of tenant rational behavior. Although this assumption is classic, it is not always verified. Tenants do not always behave rationally in the sense of being driven only by financial considerations, though commercial real estate is a factor of production. Many factors influence a decision to move such as availability of space, social impact, routine, communication, mergers and acquisitions. In some rare cases, the landlord is tempted to give a discount to short-term tenants according to elasticity of demand. Barker (2003) develops a model in which landlords consider both turnover and demand elasticity.

Even if this assumption is questionable, we do not believe it represents a major issue. Particularly in continental Europe, rent is usually in line with market rental values available for similar premises, and this is precisely because of the prospects of a break option being open to tenants. Negotiations over rent are common when break options approach. A landlord concedes financial incentives (free rent or improvements) or even rent reduction so vacancy or movement to a new, longer lease is avoided. Further research that includes utility functions would elucidate this limitation.

Another limitation is inherent to real estate as an asset class, due largely to lack of data. We implicitly suppose a normal law drives both market rental values and terminal values of the portfolio in absence of data that define a more specific law. This assumption is weak but common, but does not affect the relevance of the model. The model can be adapted by using another law or other simulation method (e.g., bootstrapping, neuronal methods, introducing extreme values etc.). The difficulty lies in a lack of data for all periods. In continental Europe, indices are generally annual (sometimes biannual), and do not cover all sub-markets in which portfolios are invested. Thus, it is difficult to calibrate a model with sparse data. The U.K. is an exception in that it provides monthly indices, but U.K. leases are characterized by length and lack break options (generally an upward-only rent review every five years). Our model is less applicable to this country even if standard leases in the U.K. change.

Another limitation is the possibility of negotiation, which is not considered in our proposal. Practitioners often negotiate when they anticipate lease breaks. Their objectives include securing future cash flows and maximizing value of the properties managed. They usually try to maintain tenants on the premises to prevent vacancy by negotiating long before a break-option. We envisage solutions to this issue. These include utility functions for a tenant or a negotiation model used in labor economics fields, modified slightly for real estate investment management. Practitioners (tenants or landlords) can also use our model as an aid to negotiation to determine the impact of a future break-option on the value of a portfolio or to determine the range of possibilities in rent negotiation, from which rent reduction is better than vacancy. We hope to propose negotiation as an addition to the model in a future research.

V. Conclusion

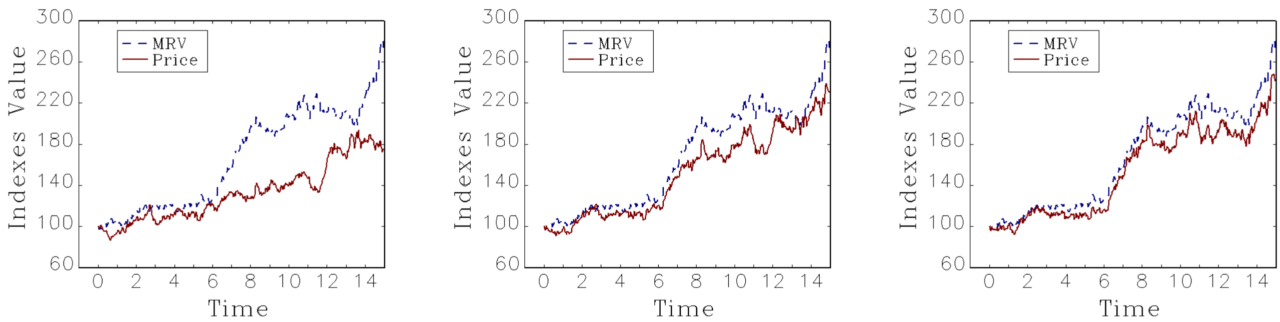
We propose a new method for valuation of a real estate portfolio using Monte Carlo simulation and options to compute price. We suggest a way of incorporating lease structure risk into the valuation process. The principal improvement in our proposal is considering break options offered to tenants in various lease structures. The model considers tenant behavior with regard to cash flows, and shows its impact on portfolio value by combining Monte Carlo methods for portfolio price under various market rental values. For each simulated scenario and for each lease at the time of a break option, we compare current rent with market rental value available for similar spaces, and assume financial rationality to decide whether the tenant moves. Poisson's law determines length of vacancy. From a practitioner's viewpoint, the model can be used to compute more robust valuations of a real estate portfolio, but reporting and risk assessment above all. Practitioners should find interesting the possibility that the model confers on displaying a histogram or a distribution function instead of a fixed value. A need for fair value of real estate portfolios remains. The problem in application is finding the inputs the model requires such as trends and volatilities of each sub-market. Our approach is flexible; many parameters can be introduced and applied to suit assumptions and needs of each investor.

This study opens the way to many other fields of real estate finance, risk management particularly. Using sensitivity analysis, we demonstrate the extent to which cash flow knowledge is of valuable assistance to risk measurement and in negotiations between landlord and tenant. Using Monte Carlo methods, we obtain a range of outcomes instead of a fixed result. Risk associated with a portfolio or a specific asset is thereby estimated better. The model, along with the distribution of its results, allows us to compute a portfolio's VaR. Both regulators and investors increasingly require such risk measures. Developing our approach in this direction - and providing robust tools for management and assessment of risk - is the objective of future research.

Appendices of chapter 1

Appendix 1:

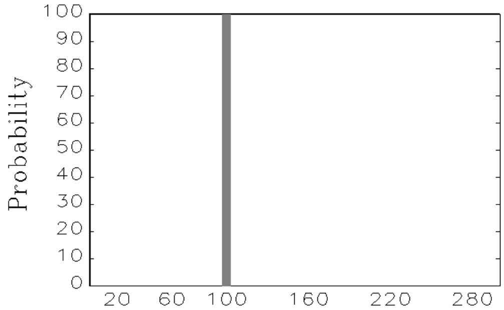
In Graph III-13, we simulate trajectories for 15 years of market rental values index $\frac{dI_{t,j}^{MRV_j}}{I_{t,j}^{MRV_j}} = 0.04\ dt + 0.08\ dW_t^{I_{t,j}^{MRV_j}}$ and of price process $\frac{dP_t}{P_t} = 0.02\ dt + 0.10\ dW_t^P$, linked by correlation $\rho_{P/I_{t,j}^{MRV_j}}$. Graph III-13(a) presents two trajectories in which correlation is not considered, and we notice a difference in trajectories. In contrast, Graph III-13(b) and Graph III-13(c) show how the correlation influences the paths.



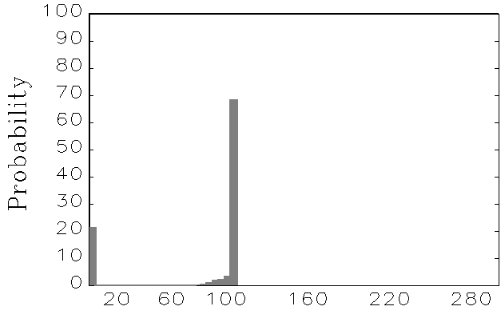
(a) $\rho_{P/I_{t,j}^{MRV_j}} = 0$ (b) $\rho_{P/I_{t,j}^{MRV_j}} = 0.6$ (c) $\rho_{P/I_{t,j}^{MRV_j}} = 0.9$

Graph III-13 - Illustration of the paths' behaviors when correlation is considered for price & market rental value

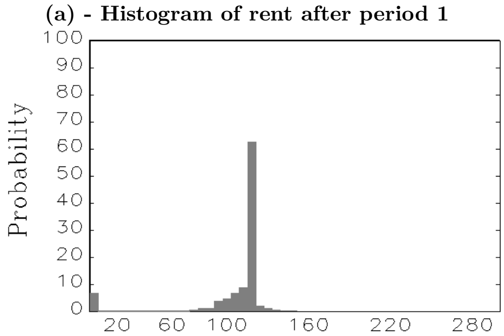
Appendix 2:



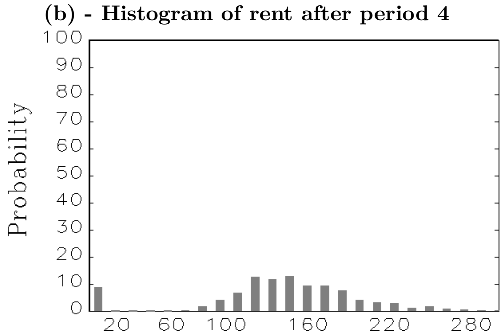
(a) - Histogram of rent after period 1



(b) - Histogram of rent after period 4



(c) - Histogram of rent after period 8



(d) - Histogram of rent after period 14

Graph III-14 - Histogram of rent in a French lease for t = 1, ...,15.

Note: In the histograms, rent is not discounted, and values on the abscissas do not have to be compared. The distribution of values remains relevant.

Graph III-14 shows the transformation of the distribution throughout a traditional French lease. It illustrates the importance of considering a break option. These options influence the distribution of values rent takes.

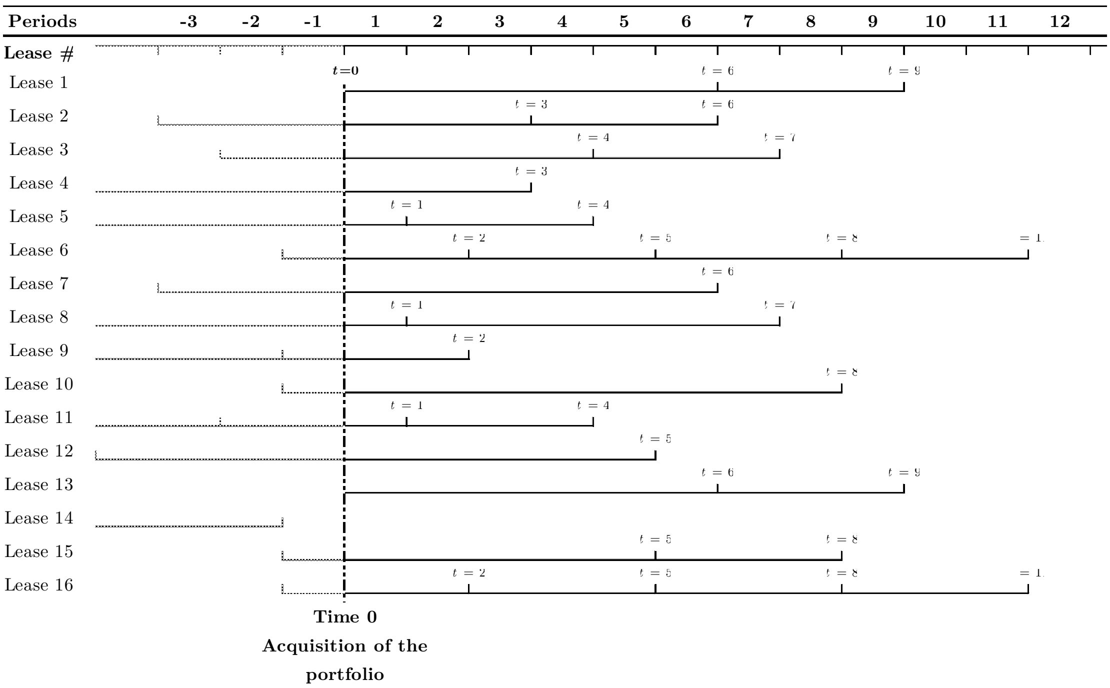
(a) - Until period 3, all rent produced are equal, and therefore the distribution concentrates at one point;

(b) - After the first break option (period 4) values primarily concentrate two points, including point 0 for all premises vacant after the exercise of the first break option, and $107.6 = 100 \times (1+2.5\%)^3$ when the tenants stay on the property. Some other points appear in the distribution, attributed to scenarios where the premises find a new tenant immediately. Thus, the transformation is visible after year 3 (at year 4). At this point, a significant part of rent is 0, representing the simulated state of the market when the space became vacant. This highlights the extent to which vacancy influences the distribution of cash flows. During the two following periods, no break options can be exercised.

(c) - This point occurs after the second break-option, if the first was not exercised. We observe an increase in the number of vacant spaces, which demonstrates some tenants exercised the option. This period exhibits the break options of tenants who contracted a lease after departure of the first tenants. The small number of tenants involved is not obvious.

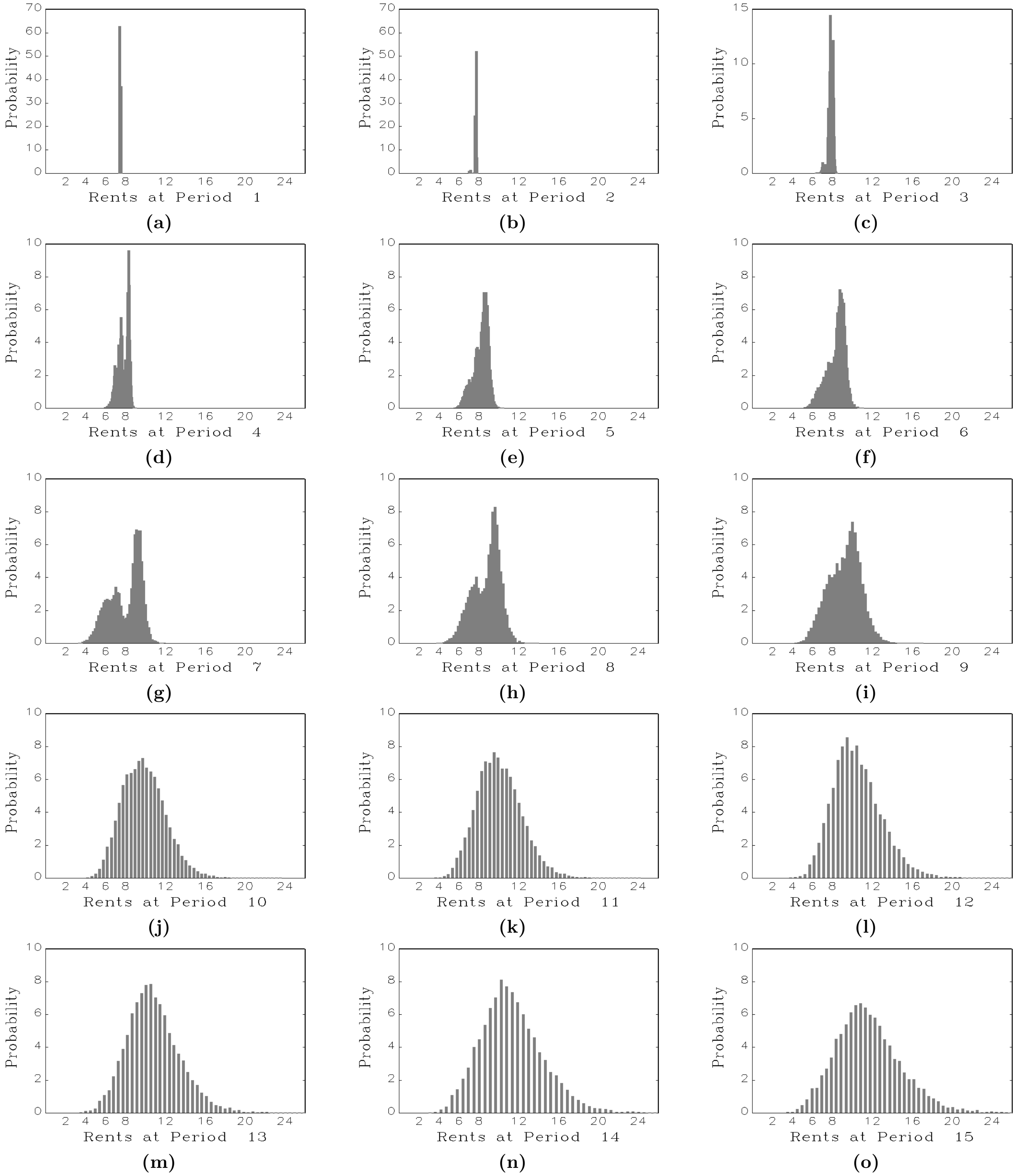
(d) - The distribution distorts after the end of the lease (year 9) when the landlord negotiates rent equal to market rental value. After year 9, all states of the market are possible because the landlord and tenant are supposed—under our assumption of rational behavior—to conclude a contract at market value.

Appendix 3



Graph III-15 - Leases during the holding period of the portfolio

Appendix 4



Graph III-16 - Histogram of rent produced by the portfolio annually over 15 years.

Comments on Graph III-16:

(a) - This is just after the first-period differences appear, due to the vacant space that can be let, or not, during the first period.

(g) - The distribution of rent produced at year 7 is noticeable and shows that this year is especially risky for the portfolio. This can be translated into action for portfolio management. For example, knowing risk, the decision could be to secure cash flows for this year either by buying portfolio insurance or renegotiating the lease in advance, selling one risky asset, or buying another one to mitigate risk in the same year. Letting and vacancy risk are therefore valued.

(o) - The distribution of rent is spread, and possibilities are multiple. It is possible to compute the probability of loss in rental level or the probability of rent falling below a specified amount.

Chapter IV. Optimal Holding Periods of a Real Estate Portfolio according to the Leases[1]

The idea for this study came from a previous paper written by Baroni et al. (2007b). That paper proposes a closed formula to obtain the optimal time to sell a real estate portfolio. Using a similar approach, we compute the optimal holding period, but we improve the previous model by adding options included in a lease. Options modify income distributions and therefore the value distribution. This work is in line with Baroni et al. (2007b) and is a companion article for the paper presented in chapter III.

This chapter deals with the holding period of a fund or portfolio; the question is a traditional one in finance. In real estate finance, this issue is not often addressed correctly since free cash flows come primarily from rent, and break-options are not usually considered. A portfolio replicates market income globally, but it has long been demonstrated that specific risk requires a very large number of assets[2] to be diversified, which is generally not attainable. Considering options to vacate in a real estate portfolio is thus fundamental. Since a diversified portfolio is difficult to obtain, idiosyncratic risk is never diversified correctly, and thus it is important to account for break options, which are one off - if not the - most important specific risks faced by real estate investors. We propose a model to determine the optimal holding period of a real estate portfolio that integrates real estate specific risk such as break options. We do not interpret the holding period as the point where value is maximized; we also consider when cash flow risks are minimized. Investors and real estate fund managers might be interested in this interpretation of optimal time to sell, which not only

[1] This article was written with Michel Baroni (Essec Business School) and Fabrice Barthélémy (University of Cergy-Pontoise). At the time, this thesis is written, the paper has been submitted to the Journal of Property investment and Finance, Emerald. In 2012, this paper was presented at an international conference, the American Real Estate Society. We received positive comments from attendees and improved the paper by following some recommendations. The author would like to thank the participants of the ARES conference who give their views and comments about the paper.

[2] See literature review part I chap. 2.

considers price risk, but also cash flow risk. Contrary to bonds that deliver recurrent cash flows, properties can be vacant (at least some units), and consequently, they generate more costs than revenues during a period. They become competitive again when the unit(s) is (are) relet (a bond that stops its debt service payment generally harbors problems), and will never reimburse the owner. The purpose of this chapter is to account for break-options in the holding period decision using our previously proposed model (chapter III of this thesis). The chapter follows extant studies Baroni et al. (2007a, 2007b), which introduce risk in the holding period decision, but do not consider break clauses included in leases.

This chapter relies on both the previously presented method and another model developed by Baroni et al. (2007b). The model proposed by Baroni et al. (op. cit.) determines the optimal holding period of a real estate portfolio. However, the model does not consider options in a lease. They use binomial law that accounts for changes in the occupancy rate of properties, and therefore considers changes in rent (the inflows). We combine option theory and Monte Carlo simulation to account for both specific and market risks, and determine the optimal time to sell a real estate portfolio. For each scenario, we determine the time that maximizes price of the portfolio while considering breaks. We then take the average of all simulated scenarios and obtain the optimal holding period. The method analyzes essential parameters that influence the holding period and determine the best lease structure for the holding period desired or given expected future market conditions.

Little research examines holding periods for real estate portfolios. However, recently, this topic was the subject of a few publications (Barthélémy and Prigent, 2009, 2011; Cheng et al., 2010a, 2010b, 2010c), but none consider vacancies in holding period decisions even if they are essential to account for specific risk. This article fills this gap by studying the effect of break-options on optimal holding period. It considers vacancies and especially concentrates on the effect of these vacancies on cash flows. This chapter involves comparison with an extant article in real estate portfolio management, Baroni et al. (2007b), which uses a similar method but does not consider options. The chapter demonstrates differences that occur when options are considered. It is demonstrated that accomplishable objectives in terms of returns and portfolio management are modified with these specifications considered. The purpose is not to predict optimal holding period, but to analyze the effect regarding its impact on the holding period of market parameters. Results of this study provide advice to real estate investors and strategists.

I. Introduction

Optimal holding period in real estate portfolio management is a recent issue. Institutional investors realized recently the importance of holding period for managing the risk of real estate assets or, more generally, real estate portfolios. For many years, real estate portfolio investment was a passive investment in which investors were buying real estate, mature asset that generated recurring cash flows (rent). The traditional strategy was to hold real estate over many years, corroborated by large transaction costs and limited liquidity of real estate investments. Few institutional investors bought opportunistic or value-added assets held for shorter periods. This typology of assets was reserved for developers, some opportunistic funds and specialized REITs. Given the specialization and sophistication of the real estate practice and industry, investors now examine the holding period of a real estate asset or portfolio as a parameter to consider when investing. The holding period is still regarded as only one parameter, dependent on either lease structure or length life of a fund instead of based on a result computed from expectations of the economy or real estate market (e.g., initial yields, future developments and rental values evolutions). Underneath this reasoning lays the criterion of liquidity, a key issue for investors. An asset leased for a long period is more liquid than a short-term leased asset, even in a bull market. Many investors look for long-term leases without break-options because they do not want to bear leasing risk. According to the local market, they look for long-term leases (e.g., 10-year leases) and sell the asset after a few years (e.g., 5 years) to an investor interested in asset management challenges, negotiations and marketing. They adapt the holding period strategy to lease structure.

Investors adjust holding period strategies in real estate based on lease structure. This can be explained through the various kinds of investors in real estate. Often for purely technical reasons, a finite holding period (conventionally 10 years in corporate finance) is used in cash flow projections to avoid an infinitely long (>30 years) cash flow series. The choice of holding period in cash flow projections must fit either the exit strategy or investors' intentions (some close-ended funds have finite lives and constrain the initial strategy). If the predefined holding period is short, the weight of terminal value in net present value is more important, and risk expected from calculation of this value is also more important. From our viewpoint, investors project to sell a property for three reasons:

- The property was managed intensively and no further asset management is envisioned. Asset managers often argue that their work consists of managing a property by securing tenants (lease and relation) and conducting work on the property such as change in property use, energy consumption, security, parking, cleaning etc).

- The asset belongs to a portfolio for which an exit strategy was defined. Depending on the strategy of portfolio management chosen by the investor, the policy regarding resale may be different. Core investors were once interested only in long-term leases without break-options, and sold all properties when the lease lengths were below a certain duration (e.g., 5 years). An opportunistic investor who seeks large capital returns might be interested only in properties that require huge asset management such as repositioning or refurbishment. The investor sells properties as soon as they are secured.
- The asset does not accurately fit the portfolio. This might be the case when too many leasing risks are concentrated on the same period or when market rental values are far lower than current rent.

All of these cases have a common point; the property is sold when the remaining lease duration is long enough to make the property attractive and liquid. Selling an obsolete, vacant property is never a goal in a business plan that optimizes portfolio value. An investor has to solve this opportunity, which occurs during the holding period. The buyer must bear vacancy and redevelopment risks, and thus might state a price that incorporates this risk (such as a large discount).

Our purpose is to consider risk linked to lease structure in future cash flows and determine how considering lease structure of a property modifies the optimal duration of a real estate portfolio. Given the behavior of a landlord in terms of holding period is driven primarily by lease structure, we model changes in holding period due to lease structure. This is achieved through a combination of Monte Carlo simulations for evolutions of terminal values and market rental values and option theory to simulate the exercising of break-options.

One of the improvements of this article is consideration of idiosyncratic risk, essential because real estate portfolios require a very large number of assets to diversify specific risk (Brown, 1998; Byrne and Lee, 2001; Callender et al., 2007). Generally, investors do not hold sufficient assets to diversify specific risk. Considering specific risk improves real estate portfolio management and, particularly, the holding period. If the market is replicated, the optimal holding period can be computed using market data alone. If the market is not replicated (generally the case in real estate), the optimal holding period must consider specific risk of the portfolio, including leasing risk as a fundamental risk.

The structure of the chapter is as follows. After a section dedicated to literature, Section 3 provides a survey concerning lease structure, with focus on European lease structures, and presents a model in which lease structure is considered in cash flows. This section serves as a brief reminder of the previous model (chapter 3). Section 4 demonstrates that the classic DCF method does not allow computing optimal holding period. Results from extant research, in which a trend is incorporated in terminal values, are also presented. Section 5 shows that the optimal holding period reacts to changes in some parameters and lease structures. Section 6 is a conclusion.

II. Literature

Real estate holding period has been an issue for real estate academicians and practitioners for a long time. It is an essential component of investment in commercial real estate portfolios. However, results concerning optimality of a holding period are nearly exclusively empirical. Presumably, it depends on many factors, including market conditions, transactions costs, types of property, lease length, strategy and others. Holding period is a classic issue in finance and has been the subject of many studies.

The issue has been studied for a long time in stock literature. Demsetz (1968) and Tinic (1972) notice transaction costs influence holding periods. Amihud and Mendelson (1986) show that assets with high bid-ask spreads (a proxy for high transaction costs) are held at equilibrium by investors who expect to hold assets for a long time. Atkin and Dyl (1997) in an empirical research consider the effects of firm size, bid-ask spread and volatility of returns on holding periods of stocks for a sample of 2000 NASDAQ firms and 500 to 1100 NYSE firms from 1981 to 1993. They demonstrate a positive correlation between holding period and transaction costs and firm size, and a negative one between holding period and price variability. Two assertions are generally accepted in stock literature: large transaction costs drive investors to hold assets for a long time and substantial volatility drives investors to hold an asset for a shorter period. Real estate assets exhibit these two features: high transaction costs and large volatility, the reason optimal holding period represents a challenge for real estate academicians and practitioners.

Real estate holding periods are the topic of many empirical studies, but no consensus emerges and the literature is sparse. For the U.S., Hendershott and Ling (1984), Gau and Wang (1994) and Fisher and Young (2000) argue holding durations are conditioned by tax laws. For the U.K., the relationship between returns and holding periods appears complex. In a study using investor interviews, Rowley et al. (1998) show investors and new property developers have a holding period in mind from the beginning. They conclude that for offices, a holding period decision links to depreciation and obsolescence factors. For retail property, the decision is empirical, depending on active management and the state of the market. In a more recent article, Collett et al. (2003) highlight knowledge of a holding period is important in a decision to invest in commercial real estate portfolios. Investment appraisal requires specifying an analysis period, and asset allocation depends on variances and covariances of assets influenced by a reference interval or analysis. Using the database of properties provided by IPD in the U.K. over an 18-year period, they conclude the median holding period is about seven years. Sales rates vary across the holding period (probably due to rent cycles and lease structures), and the holding period varies by property type. The larger and more expensive the properties, the longer the holding periods. If the return is greater, the holding period is shorter. However, they are unable to propose conclusions about a link between volatility and holding period because of absence of a proxy to measure the relationship. For small residential investments, Brown and Geurts (2005) offer an empirical response to the following questions: how long does an investor own an apartment building and why do investors sell some property more frequently than others do? Through a sample of apartment buildings of between 5 and 20 units over the period 1970 to 1990 in San Diego, they found the average holding period is approximately five years. They propose investors sell property sooner when values rise faster than rent. Brown (2004) shows that considering risk peculiar to real estate investments explains the reason for owning real estate by private investors and their buy-sell behaviors. Applying the CAPM for individuals to understand portfolio management does not drive relevant results as demonstrated by Geltner et al. (2006). For residential real estate, Cheng et al. (2010c) demonstrate higher illiquidity and transaction costs lead to longer holding periods, while higher return volatility implies shorter holding periods. These results are consistent with previous articles regarding financial assets.

Following a different approach, Baroni et al. (2007a) propose using dynamic cash flows for rent inflows and terminal values in a real estate portfolio. These dynamics are simple diffusion processes in which corresponding parameters are the trend and volatility for rent and price, respectively. These parameters are estimated from a rent index and a real estate index using Paris data, considering the correlation between these two indices. This approach suggests an interesting role played by the holding duration in the determination of asset value. Baroni et al. (2007b) determine optimal holding period when chosen initially. They model terminal values as diffusion processes, and derive a closed formula for optimal holding period under conditions of parameters. Barthélémy and Prigent (2009) also examine determination of optimal holding period (optimal time to sell in a real estate portfolio), considering knowledge the investor holds concerning the probability distribution of the real estate index. Barthélémy and Prigent (2011) provide another study on the holding period topic in real estate when risk aversion is considered.

To our knowledge, the existing academic literature on holding periods does not consider lease structure an essential decision factor. However, many investors determine a strategy as a function of the lease and not of the market or state of the economy. We propose considering the lease structure to determine the optimal holding period for a fund. The next sections introduce the optimization model and the model on which we rely to consider the leases. We start with a review of extant models for real estate portfolio holding periods.

III. Optimal holding period with traditional discounted cash flow (DCF)

Most investors originally used the DCF framework to evaluate portfolios. We demonstrate this framework is inappropriate to compute an optimal holding period for a portfolio of real estate assets. The traditional, deterministic DCF model calculates net present value that is the sum of all the future cash flows generated by the asset discounted by a discount rate. Denote $P_{0,T}$ the net present value of the asset sold at date T.

$$P_{0,T} = \sum_{t=1}^{T} \frac{FCF_t}{\left(1+k\right)^t} + \frac{TV_T}{\left(1+k\right)^T}$$

where k is usually the weighted average cost of capital (WACC) used to discount the various free cash flows FCF_t, and TV_T is the terminal value computed as:

$$TV_T = \frac{FCF_T\,(1+g_\infty)}{k - g_\infty}$$

where free cash flow (FCF) after time T grows infinitely at constant rate g_∞. If we denote g the growth rate of the free cash flows before time T, the equation becomes:

$$P_{0,T} = \sum_{t=1}^{T} \frac{FCF_1 (1+g)^{t-1}}{(1+k)^t} + \frac{FCF_1 (1+g)^{T-1} (1+g_\infty)}{(k-g_\infty)(1+k)^T}$$

Baroni et al. (2007b) demonstrate the behavior of price can be studied by computing $P_{0,T+1} - P_{0,T}$.

$$P_{0,T+1} - P_{0,T} = FCF_1 \frac{(1+g)^{T-1}}{(1+k)^T} \left(\frac{g-g_\infty}{k-g_\infty} \right)$$

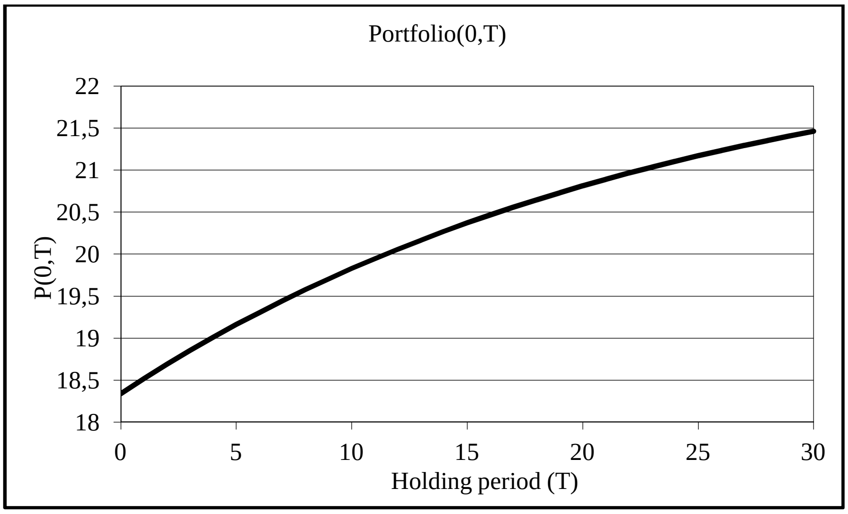
The sign on the right part of equation corresponds to the sign of $g - g_\infty$. We then have the following states:

If $g > g_\infty$ then $P_{0,T+1} - P_{0,T} > 0$ and the price grows infinitely;

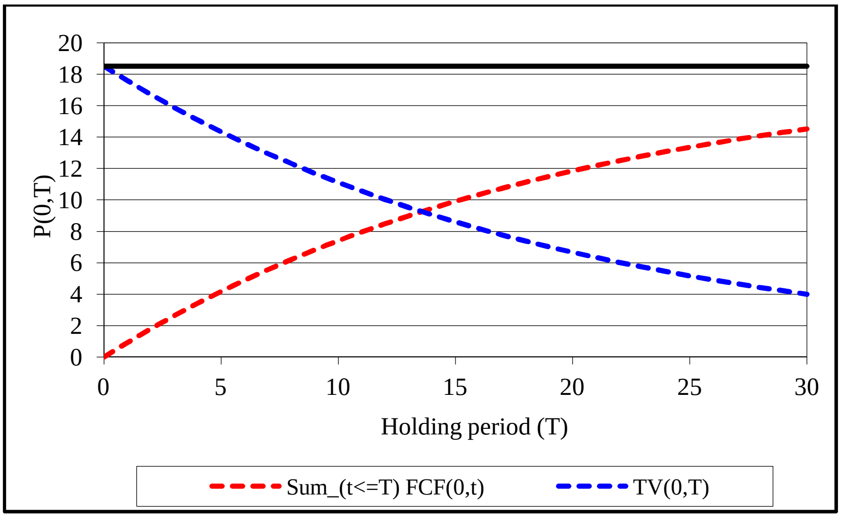
If $g = g_\infty$ then $P_{0,T+1} - P_{0,T} = 0$ and the price does not move;

If $g < g_\infty$ then $P_{0,T+1} - P_{0,T} < 0$ and the price decreases infinitely.

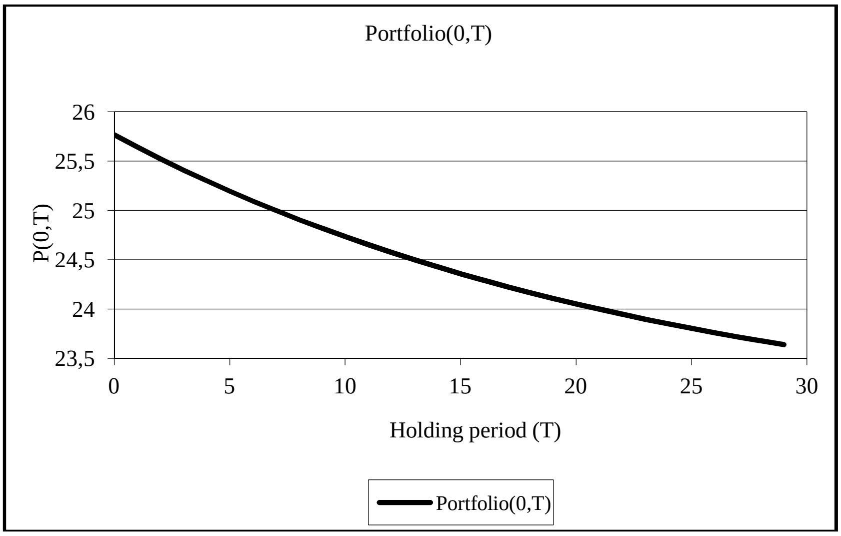
With $k = 8.40\%$, $g = 4\%$, $g_\infty = 3\%$ and $FCF_1 = 1$, Graph IV-1 illustrates the monotonous character of the function with the DCF approach. Graph IV-2 corresponds to a case where a loss in terminal value is balanced by a gain in cash flow: $g = g_\infty$. Graph IV-3 is obtained with $g_\infty = 4.5\%$.



Graph IV-1 - Increase of the portfolio present value with the DCF approach ($g > g_\infty$)



Graph IV-2 - Constant present value of a portfolio with the DCF approach ($g = g_\infty$)



Graph IV-3 - Decrease of the portfolio present value with the DCF approach ($g < g_\infty$)

The traditional DCF framework cannot allow an optimal holding period for a portfolio, according to the asset present value, whatever the rates of expected growth are.

We now introduce a model developed by Baroni et al. (2007b) that leads to a closed formula.

IV. Optimal holding period incorporating risk in the terminal value (Baroni, Barthélémy and Mokrane, 2007b)

Baroni et al. (2007a) propose Monte Carlo simulation in valuation, and their contribution is modeling terminal value. They consider the real estate price of the assets follows geometric Brownian motion (versus an infinite growth rate with traditional DCF):

$$\frac{dP_t}{P_t} = \mu_P dt + \sigma_P dW_t$$

This equation assumes real estate returns can be modeled as a simple diffusion process where parameters μ_P and σ_P are the trend and volatility. They propose this modeling to improve the DCF method to allow an optimal detention period. They then compare this new approach with the discrete case derived in the previous section.

Following Baroni et al. (2007b), the expected present value of the asset sold at date T is:

$$\mathrm{E}\left(P_{0,T}\right) = \sum_{t=1}^{T} \frac{FCF_t}{\left(1+k\right)^t} + \frac{P_T}{\left(1+k\right)^T}$$

with P_T computed using Brownian process (μ, σ) and

$$E(P_T) = P_0\left(1+\mu\right)^T$$

The expected growth rate of the price is:

$$\mathrm{E}\left(P_{0,T+1} - P_{0,T}\right) = \frac{1}{\left(1+k\right)^{T+1}}\left[FCF_1\left(1+g\right)^T + P_0\left(1+\mu\right)^T\left(\mu - k\right)\right]$$

They conclude two cases be considered:

- $\mu = k$, hence, there is no optimal holding period;
- $\mu \neq k$, an optimal sell date (under existing conditions) may exist and is obtained by a closed formula[3]:

$$T^* = \frac{\ln\left(\dfrac{FCF_1}{P_0\left(k-\mu\right)}\right)}{\ln\left(\dfrac{1+\mu}{1+g}\right)}$$

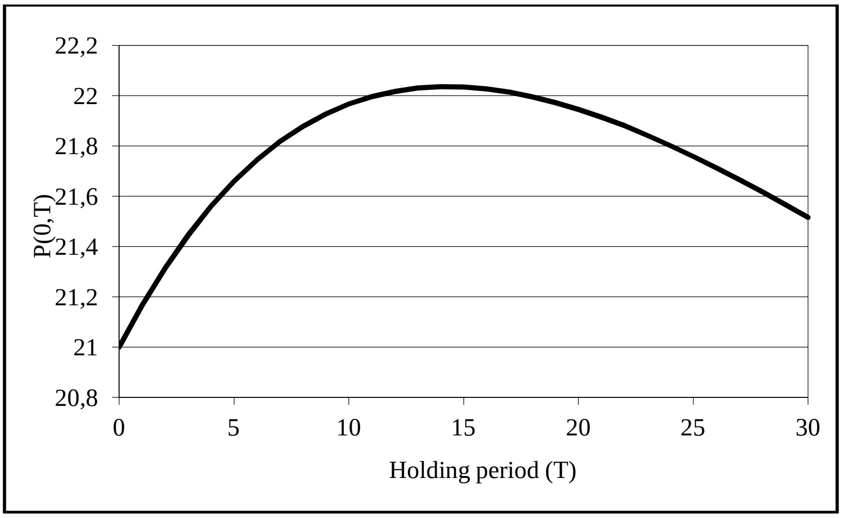
[3] The optimal holding period does not depend on the standard deviation parameter (σ_P)

This formula determines the conditions under which an optimal solution exists. The conditions can be summarized by:

$$\max\left(g, k - \frac{FCF_1}{P_0}\right) < \mu < k$$

An optimal holding period for a real estate portfolio can be thus derived.

Using the same example but simply adding a trend that fills conditions $k = 8.40\%$, $g = 3\%$, $\mu_P = 4.5\%$, $\sigma_P = 5\%$, $P_0 = 21$ and $FCF_1 = 1$, we derive an optimal holding period of about 13.81 years. Here, the free cash flows periodicity corresponds to one year.



Graph IV-4 - Optimal holding period of the portfolio value when a terminal value is simulated

Similarly, we consider risk from a terminal value and market rental value, but add a fundamental risk factor: one related to lease structure. This is possible from previous paper (chapter 3).

V. The break-option: optimal holding period incorporating risk in terminal value and lease structure (Amédée-Manesme, Baroni, Barthélémy and Dupuy, 2012)

For core investors, real estate assets are cash flow-generating assets and properties, purchased primarily for the rent they generate. One of the most essential determinants of cash flow in real estate investment (particularly commercial real estate) is the lease that stipulates the conditions under which the property is let. Regulating rights and obligations of the landlord and the tenant, the lease provides information concerning expected cash flows for years ahead. A lease specifies a starting date, initial rent, lease expiration date, indexation rules, and options available to the tenant to leave the premises before the lease expires. These last options are called break (sometimes renewal) options. They are a particular feature of continental Europe leases, and one of the most important risks European real estate investors face. In Europe, lease structures vary from country to country, and are even more different in the U.K. Information on lease structure is thus an essential component of any cash flow model. However, a property may be vacant (sometimes partially) and may generate more costs[4] than revenue. Vacancy is an essential issue for real estate investment, particularly for cash flow forecasting. Departure of a tenant is essential when valuing future cash flows.

A break option is an asymmetric right in favor of the tenant: at the time of a break option (usually fixed in the lease), the tenant has the right, but not the obligation to terminate the lease. For various reasons, a tenant may leave at the time of a break option. Therefore, break options cause vacancy and a hiatus in cash flows, and this represents the principal cash flow risk borne by investors. A vacant space incurs costs and offers no revenue. In addition, the premises may become outdated or more quickly obsolete, and thus also involves increased risk of capital loss. Previous academic studies of rental contracts such as Miceli and Sirmans (1999) suggest landlords attempt to minimize vacancy and turnover costs by offering discounts to long-term tenants. Landlords often try to dissuade a tenant from leaving at the dates fixed by the lease by offering discounts or free rent. In this way, they minimize the number of break-options (again, information concerning break clauses is reported in the general introduction to this thesis).

Lease structure risk is crucial when considering real estate investment, particularly in the context of optimal holding period. A property with a short-term lease bears more leasing risk, and therefore may be less liquid. When future cash flows are projected, considering a tenant may leave is essential, and it may have a huge impact on terminal value and the holding period. In the following model, the impact

[4] As any commodity or product traded in a free market, supply and demand control rent. If a rental property is priced above current market value, competitively priced properties rent while overpriced properties sit vacant. A property does not keep efficiency without a minimum investment. A vacant property requires substantial investment in addition to recurrent costs (e.g., local taxes, security, technical control etc.). Vacancy cost is therefore the money estimated in consequence to vacancy.

on holding period is considered by assuming terminal value is subject to systematic risk only.

A. The model

In the previous chapter (paper), we present a model that we briefly remind here. Chapter III presenting Amédée-Manesme et al. (2012) develop a model that considers lease structures (and therefore break-options) of a real estate portfolio. We improve existing commercial real estate valuation methods by introducing uncertainty and risk into the valuation process. The problem is modeling the exercise of the break-option, and in case of exercise, to determine how long a vacant space can remain vacant.[5] The model offers answers to these questions by considering risk underlying the lease structure, the risk the rent on the exercise date of the break-option is greater than the market rental value. Obviously, agency costs (e.g., moving costs, transaction costs etc.) also must be considered. Both Monte Carlo simulation and option theory to model a tenant's decision are combined to simulate future cash flows. The model incorporates uncertainty in the determination of terminal value. Both the price of the portfolio (P) and market rental values (MRV) are simulated as diffusion processes:

$$\frac{dP_t}{P_t} = \mu_P dt + \sigma_P dW_t^P$$

$$\frac{dI_t^{MRV}}{I_t^{MRV}} = \mu_{I_t^{MRV}} dt + \sigma_{I_t^{MRV}} dW_t^{I_t^{MRV}}$$

These equations assume real estate prices and real estate rental values can be modeled as Brownian diffusion process where parameters μ_P and $\mu_{I_{t,j}^{MRV^j}}$ are the price and market rental value index trends and σ_P and $\sigma_{I_{t,j}^{MRV^j}}$ the price and market rental value index volatilities. The correlation between market rental values and terminal value is also considered. Market rental value is modeled as index I_t^{MRV}, but the size of the spaces and other characteristics can be considered. Generally, two spaces located in the same property follow the same index, but differ by rent, size and specifications

[5] The space may be let or not. If let, depending on the terms of the contract, the tenant enjoys the possibility of leaving at a predetermined date during the length of the lease (the break option). When the lease terminates, both tenant and landlord decide to either continue with the lease or not, eventually with some limitations. The end of the lease is also a break option, but symmetric since both tenant and landlord can exercise it even if it implies costs. Uncertainty regarding changes in rent over time arises from the possibilities a break option will be exercised, and the length of vacancy periods

(e.g., floors, A/C system, orientation etc.). However, rent is not necessarily equal to market rental value, so the model compares rent currently paid to simulated rental values. A rational tenant exercises a break-option as soon as the rent currently paid is too high in comparison with the market rental values available for similar spaces. Therefore, the tenant leaves the space at the time of a break-option when rent is much higher than market rental value. This is written as:

$$\frac{Rent_{t,i}}{MRV_{t,i}} \geq 1+\alpha, \text{ then } Rent_{t+1,i} = 0$$

where α is a decision criteria (≥ 0 if the tenant is rational, including possible moving costs, for example), $Rent_{t,i}$ is rent of the space i at time t, and is $MRV_{t,i}$ the market rental value of space i at time t.

The model considers differences that arise between evolution of market rent and rent contracted years before a break-option. Three factors are thus considered: evolution of rent (or the ways they are revised), evolution of market rental values and evolution of possible vacancies. As soon as a tenant vacates a space, the landlord faces a void period. Length of vacancy (of the void period) is modeled using Poisson's law.

$$X \sim P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

where k is length of vacancy and λ is a positive real number equal to expected number of occurrences during a given interval. In this case, it is equal to average length of vacancy.

For each simulated scenario, we simulate a path of portfolio price and all paths for each market rental value of each unit in the portfolio (all of them being correlated). Simulated portfolio price is used in the discounted cash flow model as terminal value. For each let unit and at the time of a break-option (if any), the current rent paid (passing rent) is compared with simulated market rental value available for similar spaces. In cases where rent paid is too high in comparison to market rental value, a tenant vacates and the landlord might face a void in cash flows for the unit. Length of vacancy is determined randomly following Poisson's law, a parameter equal to average vacancy length in the market. If a space is vacant initially, length of vacancy is also determined using Poison's law. Through an assumption of player rationality (tenants and landlords), new leases are contracted at the market rental value of the unit. At each period, we calculate the price of the portfolio (simulated) and the cash flows produced by the units (some cash flows are 0 for vacant spaces). The price of the portfolio can then be computed for each holding period. We replicate the procedure thousands times and obtain a portfolio price for various holding periods. We thus obtain both averages of all scenarios and a distribution of prices for each holding period.

For clarity of presentation and assuming rational behavior of the players, we assume a new lease without vacancy at the end of a lease. In this sense, we assume landlords and tenants negotiate and conclude a new lease at the MRV.

The model is based on a stochastic approach of the classic discounted cash flow method, and incorporates both systematic (through simulation of price) and specific (structure of lease) risk for each cash flow. This approach considers a tenant's behavior regarding break options included in a lease,[6] and can be tailored to real portfolio managers to manage portfolio risk. Managing portfolio risk is also useful to determine the best time to sell a portfolio. We do this in the next section where we apply the model to determine an optimal holding period.

B. Optimal holding period

Determining an analytic formula for optimal holding period of a real estate portfolio when options embedded in the lease are considered is not possible. Following Amédée-Manesme et al. (2012), we estimate optimal holding period of a real estate fund when lease structures are considered. We consider the optimal holding period as the one that maximizes portfolio value. The model differs from the model presented by Baroni et al. (2007b) since numerous leases and introduction of break options are considered. We simulate portfolio price and incorporate risk, but rent also incorporates risk in the sense that the possibility of vacancy is considered lease by lease. Nevertheless, rent is not simulated; only market rental values are simulated.

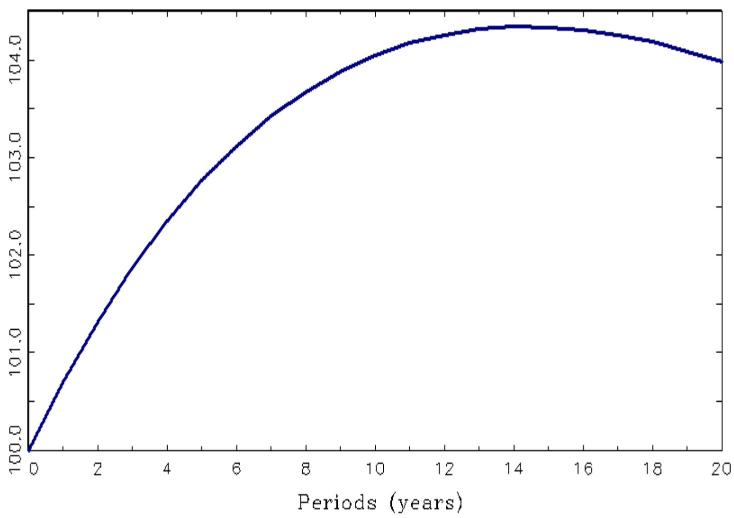
To demonstrate relevancy of the proposal and changes undergone by the optimal holding period when the lease structure is examined, we consider for a quantitative example the same parameters as do Baroni et al. (2007b); $P_0 = 100$, $FCF_0 = 4.8$, $\mu_P = 4.5\%$, $\sigma_P = 5\%$, $k = 8.4\%$, $g = 3\%$. We consider one lease structure for illustration purposes and for better understanding of the model. The portfolio holds

[6] A space may be let or not. If let, the tenant has or does not have the possibility of leaving at a predetermined date during the term of the lease (the break option). When the lease terminates, both tenant and landlord can decide to either continue with the lease or not (with some limitations, which we consider irrelevant here). The end of the lease is therefore a break option, but symmetric since both tenant and landlord can exercise it even if often with varying costs. Uncertainty regarding changes in rent over time arises from possibilities that the break option will be exercised, and length of vacancy periods.

one property (value = 100), which itself holds one lease ($Rent_0 = MRV_0 = 4.8$). In a work-in-process article, numerous properties and leases are considered. To clarify this section, we only take a simple case: 1 asset with 1 lease, otherwise the effect of the model is not easily observed. For simplicity, we consider the lease starts in the first period. We add the following parameters that refer to market rental values and the lease: $MRV_0 = 4.8$ (= FCF_0), $\mu_R = 3\%$ (= g), $\sigma_R = 0$ and $\lambda = 0$[7] and the lease structure = 3/6/9 years (nine-year lease with options at years 3 and 6). Finally, we consider the portfolio as a whole, and do not allow rebalancing or arbitrage.

We first simulate the base case and then change parameter by parameter to demonstrate the influence of lease structures. Underlined parameters are those changed from the base case or that must be noticed.

Base case: $\sigma_R = 0$: $P_0 = 100$, $FCF_0 = 4.8$, $\mu_P = 4.5\%$, $\sigma_P = 5\%$, $k = 8.4\%$, $g = 3\%$, $MRV_0 = FCF_0$, $\mu_R = 3\%$, n = 5 000, $\sigma_R = 0$, $\lambda = 0$, lease structure = 3/6/9, $\alpha = 0$



Graph IV-5 - Optimal holding period without volatility of market rental values

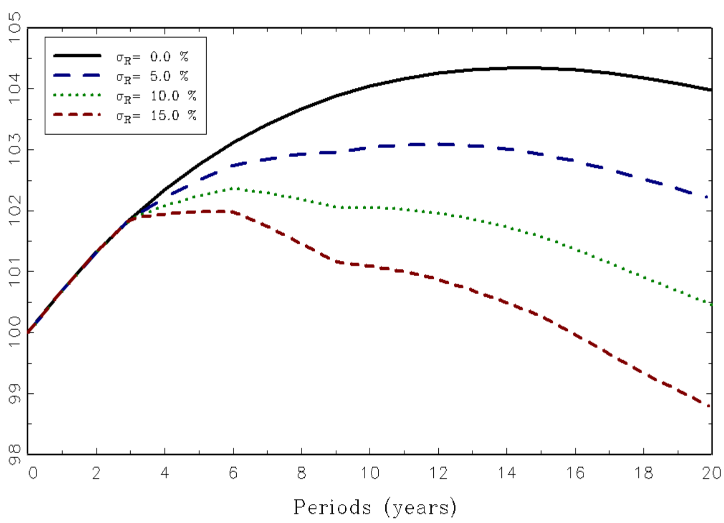
In this base case, we obtain the same results as Baroni et al. (2007b) in which rent follows a deterministic process. This is consistent with assumptions taken in this base case where there is no volatility in the evolution of the MRV:

$$\left.\begin{array}{l} MRV_0 = Rent_0 \\ g = \mu_R \\ \sigma_R = 0 \end{array}\right\} MRV_t = Rent_t$$

[7] Average lease length.

The combination of the absence of volatility and the trend of the market rental value equal to indexation leads to the same rent at each period. Therefore, no break-options are exercised since rent is never higher than market rental value. This leads to the same rent produced by Baroni et al. (2007b), and thus the same free cash flows. We obtain the same optimal holding period of 14 years. A model without volatility can be translated in a model where rent is indexed and price alone bears volatility. It is important to note that absence of options exercised makes average vacancy length meaningless.

Case 1: $P_0 = 100$, $FCF_0 = 4.8$, $\mu_P = 4.5\%$, $\sigma_P = 5\%$, $k = 8.4\%$, $g = 3\%$, $MRV_0 = FCF_0$, $\mu_R = 3\%$, n = 5 000, $\underline{\sigma_R \text{ varies}}$, $\underline{\lambda = 0}$, lease structure = 3/6/9, $\alpha = 0$



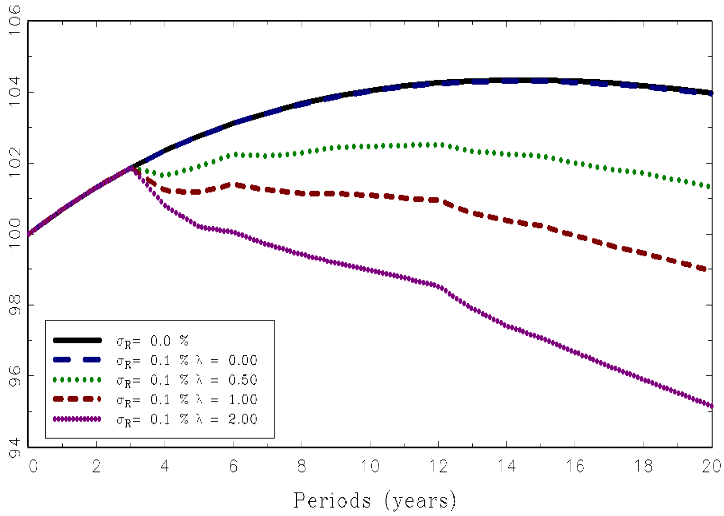
Graph IV-6 - Optimal holding period with market rent volatility

When the volatility of market rental values increases, optimal holding period decreases. The market rental value does not follow rent, and in some paths, break-options are exercised. We thus observe a decline in portfolio value due to the number of break-options exercised. At each option exercised, rent is revised to a lower value and the holding period decreases. We conclude that the optimal holding period decreases when volatility of the market rental value increases.

$$\frac{dT^*}{d\sigma_R} < 0$$

This result follows the literature concerning stocks and bonds in which holding period decreases when volatility of returns increases.

Case 2: $P_0 = 100$, $FCF_0 = 4.8$, $\mu_P = 4.5\%$, $\sigma_P = 5\%$, $k = 8.4\%$, g = 3%, $MRV_0 = FCF_0$, $\mu_R = 3\%$, n = 5 000, <u>$\sigma_R = 0.1\%$</u>, <u>λ varies</u>, lease structure = 3/6/9, $\alpha = 0$



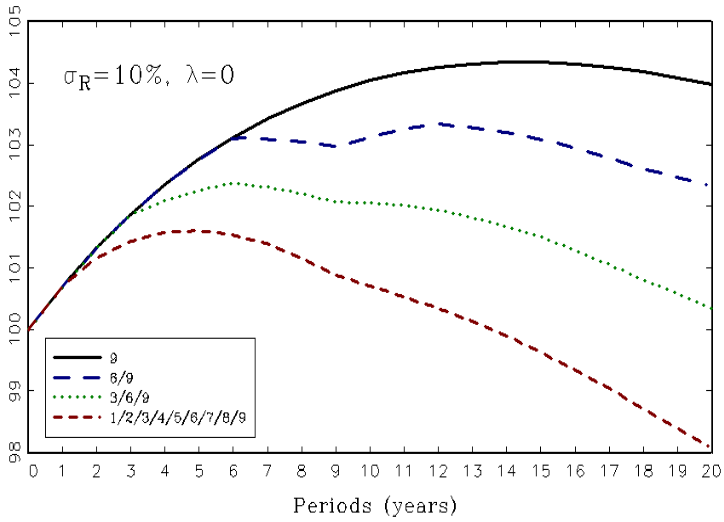
Graph IV-7 - Optimal holding period with increasing vacancy length

In the presence of low volatility and when average length of vacancy increases, the optimal holding period decreases. Length of vacancy has a huge impact on optimal holding period. This demonstrates that it is important to consider length of vacancy due to the void periods considered. These periods influence cash flows generated by the property, particularly when break-options are multiple or when the length of secured cash flows is short.

$$\frac{dT^*}{d\lambda} < 0$$

Note that the presence of a minimum volatility is mandatory. Without volatility, no options are exercised and we are back to the deterministic case presented above.

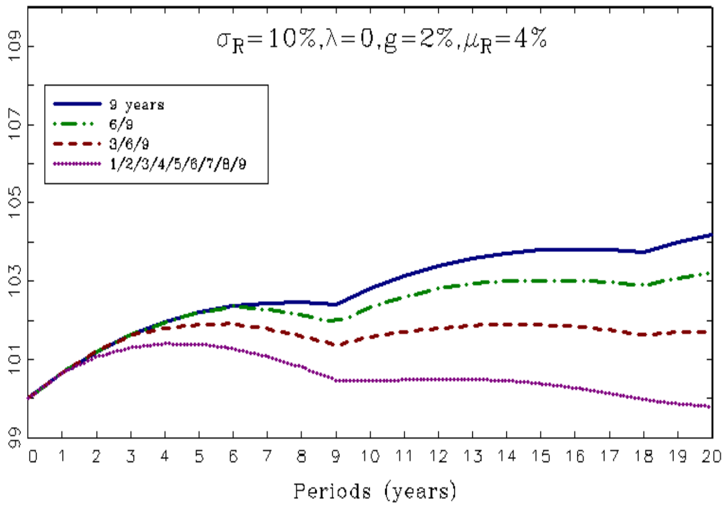
Case 3: P_0 = 100, FCF_0 = 4.8, μ_P = 4.5%, σ_P = 5%, k = 8.4%, g = 3%, MRV_0 = FCF_0, μ_R = 3%, n = 5 000, $\sigma_R = 0.1$, $\lambda = 0$, lease structure varies, α = 0



Graph IV-8 - Optimal holding period with various lease structures

The number of possible break options also has a huge impact on optimal time to sell. Length of secured cash flow makes the optimal holding period increase. Investors with a short-term lease bear more risk of void from cash flows and therefore less cash flows on average, corroborating observed behavior of contemporary investors. Generally, investors who invest in short-term, leased properties resell quickly as soon as they have released it, based on a longer lease. Contrarily, long-term leased properties are of more interest to long-term investors seeking more recurrent cash flows and minimum risk.

Case 4: g < μR: $P_0 = 100$, $FCF_0 = 4.8$, $\mu_P = 4.5\%$, $\sigma_P = 5\%$, $k = 8.4\%$, $\underline{g = 2\%}$, $MRV_0 = FCF_0$, $\underline{\mu_R = 4\%}$, n = 5 000, $\underline{\sigma_R = 0.1}$, $\underline{\lambda = 0}$, lease structure varies, $\alpha = 0$

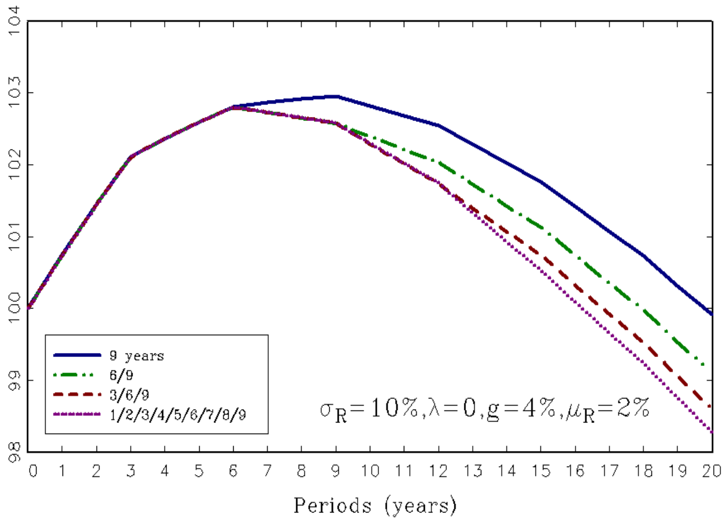


Graph IV-9 - Optimal holding period with rent growth rate lower than market rental value trend

If current rent increases at a lower average rate than the MRV, a tenant is rarely tempted to leave the premises. If rent grows slower than the market rental value, the tenant, on average, is not tempted to move. Simulation was conducted under the assumption of null average vacancy length, and therefore the few options exercised are immediately released based on lower rent. The pits observed in Graph IV-9 correspond to renegotiations at the end of each lease when the tenant and landlord concede to market rental value as rent (this is an assumption of the ABBD's model that can be modified).

A rent growth rate that is lower than the market rental value trend drives a longer holding period due to few break-options exercised.

Case 5: g > μR: $P_0 = 100$, $FCF_0 = 4.8$, $\mu_P = 4.5\%$, $\sigma_P = 5\%$, $k = 8.4\%$, $\underline{g = 4\%}$, $MRV_0 = FCF_0$, $\underline{\mu_R = 2\%}$, n = 5 000, $\underline{\sigma_R = 0.1}$, $\underline{\lambda = 0}$, <u>lease structure varies</u>, $\alpha = 0$



Graph IV-10 - Optimal holding period with rent growth rate higher than the market rental value trend

If current rent increases at a higher rate than the market rental value, tenants exercise many break-options. The investor faces many void periods, and the optimal time to sell decreases due to lower cash flows generated, on average. The market rent available, on average, is lower than rent currently paid. The tenant is therefore tempted to leave and break-options are exercised more frequently. This is consistent with observations in a bear market in which tenants move more often to reduce fixed costs, including rent.

Rental growth higher than market rental value growth drives a lower optimal holding period, due primarily to the break-options exercised.

VI. Conclusion

In this article, we compute optimal holding period of a real estate portfolio by considering possible break-options for leases in a portfolio. This is possible by relying on a model that combines Monte Carlo simulation and option theory. Obviously, market risk is considered using Monte Carlo simulation that simulates market rental value and portfolio price paths. Specific risk is considered using option theory. Options to vacate are analyzed like financial options in which the premium is not computed. We incorporate risk for cash flow (rent) and terminal value.

We conclude options embedded in leases influence optimal holding periods. The optimal time to sell a real estate portfolio depends largely on future cash flows, particularly on the security of future cash flows. We illustrate the effects of various parameters on optimal holding period. The most obvious results include: the volatility of market rental value links negatively to holding period (the more volatility, the shorter the holding period); average vacancy length links negatively to optimal holding period (the more vacancies, the shorter the holding period); the number of break options link negatively to holding period; rent growth and market rental value growth rates can be considered; rent growing faster than market rental value has more chance to be broken when possible, and this translates into a shorter holding period.

In sum, accounting for break clauses when managing a holding period of a real estate portfolio is essential. Break-options are a primary specific risk faced by portfolio managers because investors incur lack of cash flows, making optimal holding period more difficult to determine.

Many studies can be imagined following this one. One of the most important is to consider properties independently instead of the entire portfolio, and to allow selling properties at different dates to maximize returns. The portfolio may be managed actively and may sell or arbitrate properties given risks or expected risks. This may be conducted through aversion options. In addition, consideration of time on the market (properties are not liquid assets and may take months to sell) is a factor that could be the subject of research. Adding a utility function in the tenant's decision may also improve the model.[8]

[8] See for instance, Barthélémy and Prigent (2009, 2011)

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Our objective was to introduce the structure of leases and associated break-options in real estate cash flows to improve real estate portfolio management. These options are essential when a real estate portfolio is considered because specific risk cannot be diversified easily. We do this through simultaneous use of Monte Carlo simulations and option theory. Our approach considers options embedded in continental European leases drawn up with tenants who may move before the end of the contract. We combine Monte Carlo simulation for both market prices and rental values with an options model that considers a rational tenant's behavior when facing a break-option. Our findings suggest simulated cash flows that consider options are more reliable that those computed using traditional discounted cash flow methods. An inherent limitation of our model is the assumption of rationality in tenant decisions.

We analyze how options influence owners' income. Practitioners can use this model to check relevancy of an investment or determine portfolio risks. Debt service issues can be addressed using this approach particularly concerning distribution of outcomes. The model allows us to analyze the effects of break-options on optimal holding period. We improve on an extant study from Baroni et al. (2007b) by incorporating break clauses in the decision process, which is both essential for reliability and in line with practice.

Given many recent regulations and growth in investor interest, risk of loss is again considered. A real estate portfolio cannot be managed correctly without estimating extreme losses. These possible losses are now considered when an investment decision must be made. Now, VaR must be computed for capital adequacy, a topic we address in the next part.

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PART III – REAL ESTATE RISK MANAGEMENT: VALUE AT RISK

The third part of the thesis focuses on risk management, and we concentrate on risk measurement: Value at Risk (VaR). Risk management is the process of identifying and analyzing uncertainty in investment decisions over the life of an investment. This last part - the life of the investment - is particularly important in real estate. Property investment is not passive; it is active in which decisions must be made, sometimes taking time to apply. It consists of quantifying potential losses and taking appropriate action given investment objectives and risk tolerance. Poor risk management results in losses for both companies and individuals involved in the investment.[9] The risk management process requires first determining what risks exist in an investment, and second handling those risks to fulfill investment objectives. Risk management is an essential part of financial management. It is mandated by regulation, and in practice, risk measures such as VaR are ways to gauge risk.

We pay particular attention to VaR in this part of the thesis. VaR is a technique used to estimate the probability of losses and is based on statistical analysis of historic prices or returns. Banks, insurance companies, security firms and other companies involved in investment management use VaR. Its calculus is now standard in all annual reports. Until recently, this calculus was not standard in real estate finance, and its computation is still infrequent due primarily to sectors that are complex and opaque.

Traditional methods to compute VaR are presented in the introduction of this thesis, but these methods suffer from many constraints when applied to real estate finance. Lack of reliable data makes it difficult to determine distributions of returns and estimate parameters required by models. Characteristics of property investments such as limited liquidity, tangibility, location, obsolescence and investment size yield VaR models created for stocks and bonds inadequate for real estate investments. This is why we concentrate on real estate VaR in this part of the thesis.

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[9] For example, loose credit risk management by financial firms largely caused a recession that began in 2008.

The first chapter concentrates on a VaR model for non-normal distributions by mixing two methods: Cornish-Fisher expansion and rearrangement procedure. This way, the quantile function of the distribution can be determined reliably and VaR computed quickly. The second chapter concentrates on a VaR model that considers characteristics of property investment. Specific and market risks are used to compute VaR specific to a portfolio.

The two chapters presented in this part of the thesis have led to two papers:

Chapter V: Paper 3: Cornish-Fisher expansion for real estate Value at Risk

Chapter VI: Paper 4: Value at Risk: a specific real estate model

Chapter V. Cornish-Fisher[1] Expansion for Real Estate Value at Risk[2]

This study was initiated following a conference hosted by the INREV association (a real estate business association) concerning the ways Solvency II treats real estate. Non-normality of real estate returns was not considered in VaR computations, piquing our interest. We faced an issue of validity and realized Cornish-Fisher was rarely applied to real estate finance. However, we searched for solutions and found a paper from Chernozhukov et al. (2010) that demonstrates how rearrangement procedures solve the issue.

Real estate returns are known to be distributed non-normally. Available databases are less than sufficient; better markets exhibit more than 250 data points. This is highlighted in the introduction of this thesis. For these reasons, traditional models used to compute VaR are unsuitable. These models suffer either from strong assumptions such as normality of returns for Monte Carlo and parametric methods or from lack of data from the real estate sector. Computation of VaR at the 0.5% level as requested by the Solvency II framework requires a minimum of 200 values. Even with 200 values, the minimum of the series might be an outlier. Therefore, it is necessary to rely on other models to compute VaR. Problems include difficulty in identifying the distribution of a series and dealing with lack of data from the real estate sector. We propose a model that neither relies on strong assumptions nor requires too much data.

[1] See Cornish and Fisher (1938)

[2] Fabrice Barthélémy (University of Cergy-Pontoise) and I started the initial work. Donald Keenan (University of Cergy-Pontoise) joined the team to give a final hand. The paper has international interest even if the application is in the U.K., and we hope to submit it to a good journal such as Journal of Real Estate Finance and Economics or Real Estate Economics, two American-ranked journals. We will submit it in the weeks following the defense. In 2012, the paper was presented at the European Real Estate Society's annual conference in the PhD paper session category and was awarded most commended paper. We received strong positive comments to continue research in this area. The authors would like to thank participants of the ERES conference and in particular Peter Byrne for their comments and advices.

Cornish-Fisher combines with rearrangement procedures well because it requires neither assumptions on return distributions nor large data sets.

The primary innovation of this method is in mixing two extant theories: Cornish-Fisher expansion and rearrangement procedures. Cornish-Fisher expansion suffers from a significant issue; it does not preserve monotony of a distribution, particularly for the lowest quantiles where quantiles are necessary for VaR computation. We apply a regulating procedure called rearrangement to monotonize Cornish-Fisher expansions, an approach Chernozhukov et al. (2010) develop and demonstrate. Following this idea, we propose a new way to compute VaR for real estate distributions. The technique is robust for many distributions and allows us to determine more reliably the quantile function of distributions. We apply the method to a number of distributions that differ from a Gaussian distribution, and illustrate it. We also illustrate our method using the U.K. IPD all-property capital returns database and compare results with those obtained from regulators of the Solvency II framework, calibrated using the U.K. IPD all-property, total-return database for the standard model.

We examine the benefits of a VaR computation model that considers non-normal distributions. This kind of model has not been used in direct commercial real estate finance even though the literature highlights non-normality of return distributions. We contribute to extant literature by adding a new method to compute VaR in real estate. Our approach is possible due to a combination of rearrangement procedure and Cornish-Fisher expansion. We apply this model to the IPD U.K. database and demonstrate that standard approaches underestimate VaR in the U.K. market. The purpose of this article is not to predict VaR in a real estate market, but to compare the proposed approach with the standard normal-assumption method. Results of this study demonstrate that Solvency II regulations calibrated by the IPD U.K. database do not assess risk taken on by real estate investors correctly. The results provide insights into the realities of risk undertaken by property investors and information concerning the reliability of standard models created by regulators.

I. Introduction

The stock market crashed in 1987, triggering new risk-measures development. This was the first major financial crisis in which practitioners and academicians were concerned about global bankruptcy. The crash was so improbable to happen given standard statistical models that all quantitative analysts cast doubts and began to question the models. Many academicians claimed the crisis would recur and called for reconsidering models. Considering extreme events had become obvious. Limitations of traditional risk measures were recognized, and measuring the risk of falling asset values was becoming urgent. Relying on a risk measure that considered the entire return distribution of a portfolio was necessary. Throughout the 1990s, a new risk measure was developed: Value at Risk, with acronym VaR.[3]

The first step of VaR was worldwide adoption of the Basel II Accord in 1999, with near completion today (Basel III must be applied by 2019). The Basle committee requires banks to compute VaR periodically and maintain sufficient capital to pay losses projected by VaR. Unfortunately, there is more than one measure of VaR since volatility, a fundamental component of VaR, is latent. Therefore, banks must use many VaR models to compute a range of prospective losses. More recently, Solvency II regulations (for insurers in Europe) proposed VaR as a reference measure to determine required capital.

In this chapter, we discuss neither VaR quality as a risk estimator nor adequacy of the measure for risk budgeting purpose. Regulators chose VaR for required economic capital calculations, and its computation is mandatory for all regulated practitioners. VaR is an essential research field that should be of interest to academicians.

Many studies concentrate on the best methods to compute VaR. We point out three articles presented in the literature review and highlight other articles focusing on Cornish-Fisher. Pichler and Selitsch (1999) compare five VaR methods in the context of portfolios that include options: Johnson transformations, Variance-Covariance, and three Cornish-Fisher-approximations for the second, fourth and sixth orders. They conclude that a sixth-order Cornish-Fisher approximation is best approaches compared to the other approaches. Mina and Ulmer (1999) compare Johnson transformations, Fourier inversion, Cornish-Fisher approximations, and Monte Carlo simulation, concluding Johnson transformations are not a robust choice. Monte Carlo and Fourier inversion are robust, and Cornish-Fisher is fast but a bit less robust, particularly when the distribution is far from normal due to non-monotony of a Cornish-Fisher expansion. Feuerverger and Wong (2000) focus on when to use Cornish-Fisher in comparison to Fourier inversion, saddle point methods, and Monte Carlo. The paper concludes with an extension of the method, which includes higher-order terms. Jaschke

[3] In our context, VaR is computed for a static portfolio with no changes in structure, and no trading or arbitrage.

(1999)[4] concentrates on Cornish-Fisher properties and underlying assumptions in the context of VaR, with particular focus on monotony of the distribution function and convergences that are not guaranteed. Jaschke discusses how these assumptions make Cornish-Fisher appear undesirable and difficult to use. However, he demonstrates that when a dataset fits required assumptions, accuracy of Cornish-Fisher expansion is generally more than sufficient, in addition to being faster.

Proper use of Cornish-Fisher expansion should avoid one important pitfall; the formula is valid only if skewness and kurtosis coefficients belong to a particular set. In practice, this constraint is not verified often (e.g., when the excess of kurtosis is negative). Chernozhukov et al. (2010) propose a procedure called increasing rearrangement to remedy the issue, and we apply this method to the real estate sector.

VaR has been the subject of some articles, but these papers focus on listed real estate and not direct real estate. VaR for listed real estate relies on the same methods as that used for stocks or bonds. Zhou and Anderson (2012) concentrate on extreme risks and behavior of REITs in abnormal market conditions. They found no universal method for VaR in listed real estate and estimation of the risk for stocks and REITs requires different methods. Cotter and Roll (2011) study REIT behavior over the past 40 years, highlighting the non-normality of REIT returns. They compute VaR of the index following three methods that do not rely on Gaussian assumptions, including the Efficient Maximization algorithm, the Generalized Pareto Distribution and a GARCH model. Liow (2008) uses extreme value theory to assess VaR dynamics of ten major securitized real estate markets. Extreme value theory allows the author to consider stochastic behavior of the tail. Extreme market risks are assessed better than when using traditional standard deviation measures, and real estate forecasts are more accurate.

Literature focusing on VaR in the context of direct real estate investment (or funds) is sparse. Numerous studies concentrate on risk management and assessment in real estate. Booth et al. (2002) examine risk measurement and management of real estate portfolios, suggesting practical issues force real estate investors to treat real estate differently from other asset classes. They particularly highlight that direct real estate may be an area for further research. The report is a complete review of the range of risk measures used to assess real estate risk. It focuses on the difference between symmetric measures such as standard deviation and downside risk measures such as VaR. Their work concentrates on all risk measures usable in real estate, and their purpose is not to propose innovation, but to convey a survey of real estate risk

[4] See also the chapter (by Jaschke and Jiang) of Härdle et al. (2008) for a detailed presentation.

measures. Gordon and Wai Kuen Tse (2003) consider VaR a tool to measure leveraged risk in the case of a real estate portfolio. Debt in a real estate portfolio is a traditional issue studied in real estate finance. The paper demonstrates that VaR allows better assessment of risk. Particularly, traditional risk-adjusted measures (e.g., Sharpe ratio, Treynor's and Jensen's alpha) suffer from a leverage paradox. Leverage adds risk along with potential for higher returns per unit of higher risk. Therefore, the risk/return ratio does not change noticeably and is not an accurate tool to measure risk inherent in debt. Contrarily, VaR is a good tool for leveraged risk. Brown and Young (2011) focus on a new way to measure real estate investment risk called spectral measures. They begin by refuting the assumption of normally distributed returns that flaw forecasts and decisions. The nature of risk and how it should be measured is discussed. Interestingly, VaR is not retained, and expected shortfall is recommended more highly. The authors focus on spectral measures, their recommendation.

From our knowledge, the use of Cornish-Fisher expansion to determine VaR in real estate has not been a subject in much literature. No study focuses solely on Cornish-Fisher in a real estate context. Lee and Higgins (2009) use Cornish-Fisher expansion in a real estate context, arguing that the Sharpe performance formula neglects two important characteristics of real estate returns: non-normality and autocorrelation. They apply Cornish-Fisher expansion to adjust Sharpe ratio performance to non-normality. Farelly (2012) presents a study that focuses on measuring risk of an unlisted property fund using a forward-looking approach. Among other relevant results, the author considers moments' measurements of orders higher than two (asymmetry considered) using Cornish-Fisher expansion. Following these authors, we motivate our research on needs of better VaR assessments in the direct real estate field.

Research on risk focusing on direct real estate or unlisted vehicles is scarce in spite of increasing interest, due likely to both lack of data from the commercial real estate sector and issues arising from non-normality of returns. Limited data from the sector is one of the primary obstacles of reliable VaR computations. Either you invest in listed real estate and it is quoted daily and sufficient data are available to compute VaR for your portfolio, or you invest in direct real estate and you deal with small datasets. This is particularly true in commercial real estate in which institutional investors largely invest. The real estate market is comparable to the private equity market where indices are created from small numbers of transactions. A real estate property index attempts to aggregate real estate market information to provide a representation of underlying real estate performance. However, this is generally conducted monthly in the best cases, quarterly or semi-annually, or sometimes annually. This links largely to sector. The residential field, where many transactions are observable, exhibits a monthly index frequently. Commercial real estate (e.g., offices, shopping centers etc.) faces more difficulties to deliver indices, with large periodicity. To determine VaR of a real estate portfolio at threshold 0.5% (as requested by Solvency II framework) using the historic approach, a minimum of 200 values is needed, which represents 17 years of a monthly index. With that number of observations, VaR is irrelevant since it corresponds to the minimum of a series. Hence, it is necessary to use other methods to determine VaR for direct commercial real estate.

The non-normality of real estate return distributions is a perplexing issue for VaR computation. This point has long been demonstrated by Myer and Webb (1994), Young and Graff (1995) and Byrne and Lee (1997). Recent studies such as Lizieri and Ward (2000), Young et al. (2006) and Young (2008) show that real estate returns usually exhibit non-normal returns. These works focus mainly on Anglo-Saxon economies, but similarities in real estate return distributions are assessed. Real estate returns skew left and exhibit fat tails. The distribution used to estimate VaR of a portfolio is determined from returns distribution or corresponding sector indices. Nevertheless, the normality assumption is adopted regularly to determine VaR because it allows quick and easy computation.

Both lack of data and non-normality issues must be considered when determining real estate VaR and this motivate our study. The case of Solvency II regulation (European regulation for insurers) is particularly interesting. They base capital requirements on VaR estimation, and propose either a standard or an internal model. The standard model for real estate VaR leads to required capital of 25% for real estate investments. This calculation was made on IPD U.K. all properties total return index, which applies to the U.K. only even if the regulation concerns all of Europe. This index is one of the only reliable commercial monthly indices in Europe. The committee recognizes the non-normality of real estate returns, but do not estimate real estate required capital. They also identify lack of data and the difficulties of computing VaR. In the absence of better datasets and indices, they recognize imperfections in their recommendations. Nevertheless, they admit an historic VaR of 25% for all European countries computed using a U.K. monthly total return index, leaving further research for internal models. Their result, as they highlight, is similar to the one obtained with a Gaussian assumption. Following these observations, we seek VaR methods that consider non-normality of real estate returns in VaR computations, without relying on large datasets. This is what Cornish-Fisher expansion combined with rearrangement procedures does. The expansion uses moments of orders higher than two, and therefore deals with non-normal distributions. The Cornish-Fisher approximation transforms a Gaussian quantile according to skewness and kurtosis coefficients.

In spite of growing needs for better VaR assessment, due to regulatory requirements and extreme risk estimations, research on the topic remains scarce. To estimate VaR of direct real estate and unlisted property funds, we use Cornish-Fisher expansion and a rearrangement procedure (Chernozhukov et al. 2010). This method explicitly accounts for asymmetry and fat-tail characteristics of direct real estate returns.[5] We improve both traditional and regulators' model (i.e., standard models). This research contributes to extant literature by employing a method that is not based solely on the first two statistical moments, and has pertinence given the current regulatory environment; recent regulations base capital requirements on VaR.

The remainder of the chapter is organized as follows. Section 2 introduces the Cornish-Fisher expansion and discusses technical points, and Section 3 implements the model. Section 4 discusses limitations, and section 5 concludes.

II. Variance-Covariance Value at Risk and Cornish-Fisher adjustment

After a brief review of VaR in the Gaussian case, we analyze Cornish-Fisher expansion and its implications on VaR computations.

A. VaR with Normal assumption:

If the returns are supposed to be normal, we would know the quantiles of a distribution. Let X be a random variable that models return value $X \sim N\left(\mu, \sigma^2\right)$. This random variable can be written as a function of standard normal variable Z as follows: $X = \mu + Z\sigma$. We define z_α, the standardized Gaussian quantile, at threshold α, which verifies $F_Z(z_\alpha) = \alpha$. Corresponding quantile q_α for X, which corresponds to $F_X(q_\alpha) = \alpha$, can be written as:

[5] VaR calculated using Cornish-Fisher expansion modifies multiplicity associated with normal law to consider moments of orders higher than two for the return distribution.

$$\forall \alpha \in \left]0,1\right[, \; q_\alpha = \mu + z_\alpha \sigma \qquad (1)$$

Suppose z_α and q_α are quantile functions. The VaR at threshold α for investment I_0 is equal to[6]

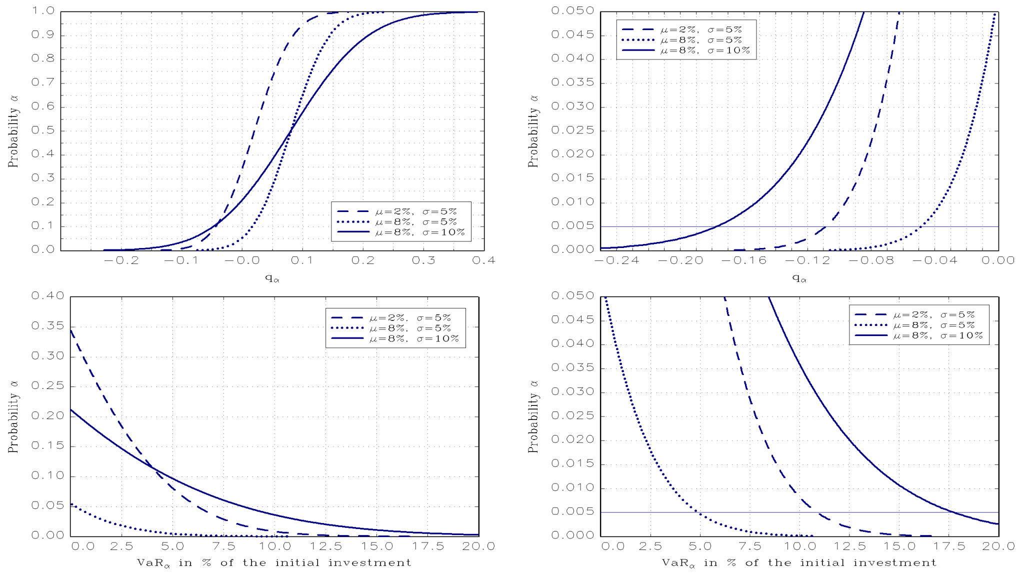
$$\forall \alpha \in \left]0, F_Z\left(\tfrac{-\mu}{\sigma}\right)\right[, \; VaR_\alpha(I_0) = -q_\alpha \times I_0 \qquad (2)$$

q_α may be interpreted as the VaR in percentage, that is relative to investment:

$$\frac{VaR_\alpha(I_0)}{I_0} = \frac{100 \times -q_\alpha \times I_0}{I_0}\% = -100\, q_\alpha \% = 100\, q_{1-\alpha}\% \qquad (3)$$

To analyze whether both μ and σ influence VaR, consider three sets of parameters for a Gaussian distribution: $X_1 \sim N\left(0.02, 0.05^2\right)$, $X_2 \sim N\left(0.08, 0.05^2\right)$ and $X_3 \sim N\left(0.08, 0.10^2\right)$. The first two graphs on Graph V-1 represent both cumulative distribution functions for these three cases, with focus on probabilities lower than 5%. The increase of the expectation from case 1 to 2 (from 0.02 to 0.08) leads to a higher 0.5% quantile (i.e., less negative); -0.0488 instead of -0.1088. This implies a lower VaR, around 73.19 instead 163.19, for the same initial investment of 1,500. An increase of the standard deviation from case 2 to 3 (from 0.08 to 0.10) leads to a lower 0.5% quantile (i.e., more negative); -0.1776 instead of -0.1088. This implies a higher VaR, around 266.37 instead 163.19, for the same initial investment of 1,500. As shown in relation (3) and presented in Table V-1, the computation of the VaR relative to the investment is simply the quantile in percentage.

[6] Since $q_\alpha = -q_{1-\alpha}$, we have $VaR_\alpha(I_0) = q_{1-\alpha} \times I_0$. Since VaR is a loss, this concept exists for a given α if and only if q_α is negative. Hence, $q_\alpha < 0 \Leftrightarrow \mu + z_\alpha \sigma < 0 \Leftrightarrow \alpha < F_Z\left(\frac{-\mu}{\sigma}\right)$.



Graph V-1 - Quantile function and VaR for a Gaussian distribution as a function of μ and σ

	Case 1	*Case 2*	*Case 3*
μ	*2%*	*8%*	*8%*
σ	*5%*	*5%*	*10%*
$q_{0.005}$	*-0.1088*	*-0.0488*	*-0.1776*
$VaR_{0.005}(I_0=1500)$	*163.19*	*73.19*	*266.37*
$VaR_{0.005}(I_0=1)$	*10.88%*	*4.88%*	*17.76%*

Table V-1 - 0.5% quantile and VaR for a Gaussian distribution as a function of μ and σ

B. VaR with quasi-normal assumption: Cornish-Fisher expansion

Cornish and Fisher (1937) developed the Cornish-Fisher expansion. In a case of independent and identically distributed random variables, it is possible to obtain explicit expansions for standardized quantiles as functions of corresponding quantiles of a unit's normal distribution. The terms of these expansions are polynomial functions of the corresponding unit's normal quantile.[7] Coefficients of the polynomials are functions of the moments of the distribution with which we are dealing. We denote z_α and $z_{CF,\alpha}$ as the Gaussian and the Cornish-Fisher quantiles, respectively. Following the presentation of Chernozhukov et al. (2010), we obtain the following approximation at order 3:[8]

$$\forall \alpha \in \left]0,1\right[,\ z_{CF,\alpha} \simeq z_\alpha + \frac{1}{6}\left(z_\alpha^2 - 1\right)S + \frac{1}{24}\left(z_\alpha^3 - 3z_\alpha\right)(K-3) - \frac{1}{36}\left(2z_\alpha^3 - 5z_\alpha\right)S^2 \qquad (4)$$

Where S and K denote skewness and kurtosis coefficients, respectively. The corresponding modified Cornish-Fisher quantile then is:

$$\forall \alpha \in \left]0,1\right[,\ q_{CF,\alpha} = \mu + z_{CF,\alpha}\sigma \qquad (5)$$

and VaR is

$$\forall \alpha \text{ such as } q_{CF,\alpha} < 0,\ VaR_{CF,\alpha}(I_0) = -q_{CF,\alpha} \times I_0 \qquad (6)$$

Mentioned previously, for z_α and q_α, $z_{CF,\alpha}$ and $q_{CF,\alpha}$ are quantile functions.

Although the Cornish-Fisher expansion is a useful tool, there are limitations for values of moments that lead to proper cumulative distribution functions (cdf). Relation (4) implies a non-monotonic cdf (i.e., the distribution's quantiles order are not preserved), violating a basic condition associated with proper cdfs. Barton and Dennis (1952), Draper and Tierney (1971) and Spiring (2011), among others, provide quantitative solutions to study the domain of validity for the Cornish-Fisher expansion. Monotony implies the derivative of $z_{CF,\alpha}$ relative to z_α is non-negative. This leads to the following constraints, which define implicitly the domain of validity (D) of the Cornish-Fisher expansion:[9]

[7] This approximation is based on the Taylor series developed by Spiring (2011). Stuart and Ord (1994) and Stuart et al. (2009) report details on the expansions.

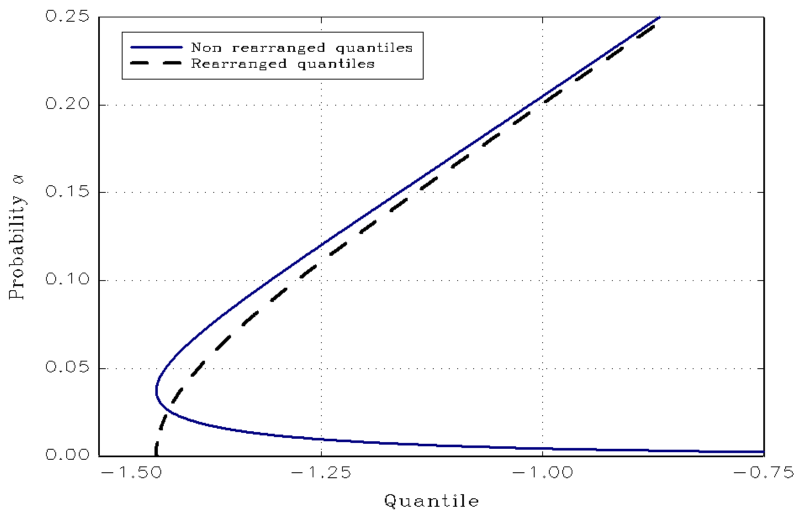
[8] At second order, the approximation is: $\forall \alpha \in \left]0,1\right[,\ z_{CF,\alpha} \simeq z_\alpha + \frac{1}{6}\left(z_\alpha^2 - 1\right)S$.

[9] For example, inequality implies a kurtosis coefficient higher than 3 (a positive excess of kurtosis), which corresponds to leptokurtic distributions. Thus, Cornish-Fisher is not relevant for platokurtic distributions.

$$\frac{S^2}{9}-4\left(\frac{K-3}{8}-\frac{S^2}{6}\right)\left(1-\frac{K-3}{8}-\frac{5S^2}{36}\right)\leq 0 \quad (7)$$

If $(S,K)\in D$, then the quantile function is monotonic. If not, the Cornish-Fisher expansion is irrelevant. Nevertheless, Chernozhukov et al. (2010) propose a procedure called rearrangement to restore monotonicity. The procedure corresponds to a sort of the function of interest. As they mention and demonstrate, rearrangement pushes non-monotone approximations closer to the true monotone target function. In our problem, it corresponds to ascending sort of the quantile function $q_{CF,\alpha}$.

Consider skewness of 0.8 and kurtosis of 2, excess kurtosis equal to -1. These parameters correspond to a platokurtic and right-skewed distribution function. Since these parameters do not belong to the domain of validity, the $z_{CF,\alpha}$ quantile function is not monotonic. Applying rearrangement procedure to this quantile function, we obtain $\tilde{z}_{CF,\alpha}$, the corrected Cornish-Fisher transformation of the Gaussian quantiles. Focusing on probabilities less than 25%, Graph V-2 shows the impact of this procedure.

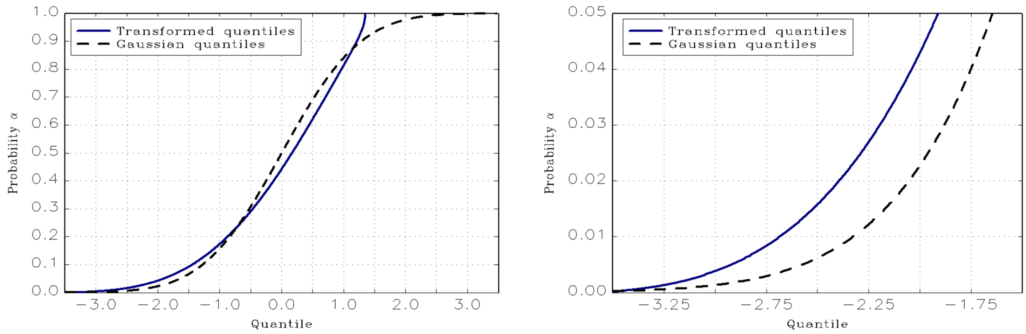


Graph V-2 - Rearrangement procedure $(\alpha < 0.25)$

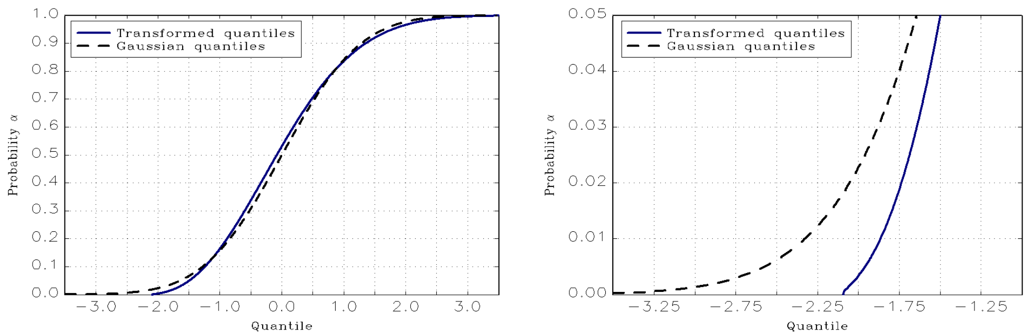
Discrepancy between the two quantile functions is noticeable for the smallest probabilities, which are the most important ones for VaR computation. The non-rearranged quantile function may be more severely non-monotone (and therefore could provide poorer approximations of the distribution function) in comparison to the one presented in Graph V-2[10]. We note $\tilde{z}_{CF,0.001} = -1.4$, while $z_{CF,0.001}$ is clearly biased and equal to -0.3.

We illustrate Gaussian quantile modification on the distribution according to skewness and kurtosis coefficients. Four cases are considered, each departing from the Gaussian reference:

- a negative and positive skewness, and the kurtosis coefficient equal to 3 (Graph V-3)
- a negative and positive excess kurtosis, and the skewness coefficient equal to 0 (Graph V-4)



a) Left-skewed distribution: $S = -1$ and $K = 3$



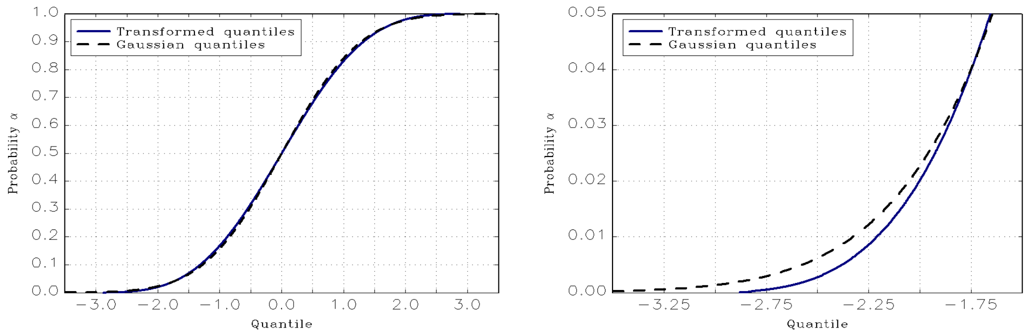
b) Right-skewed distribution: $S = 0.5$ and $K = 3$

Graph V-3 - Cornish-Fisher and normal quantiles according to the skewness coefficient

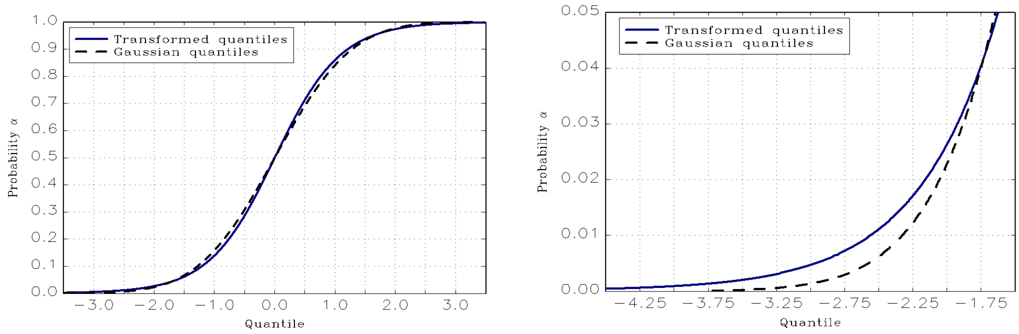
For each example, $\tilde{z}_{CF,\alpha}$ and z_{α} quantile functions are represented. This is done for the entire function and to focus on the smallest values of α. For a left-skewed distribution (Graph V-3a), the smallest quantiles are lower than the Gaussian ones. This is contrary to a right-skewed distribution (Graph V-3b). Thus, VaR is higher for a left-skewed distribution than for a Gaussian one, and lower for a right-skewed distribution.

[10] See the first figure presented in Chernozhukov et al. (2010).

For instance, a skewness coefficient of -1 leads to $\tilde{z}_{CF,0.05} = -1.9103$, while $z_{0.05} = -1.6449$; $\tilde{z}_{CF,0.001} = -3.3049$ and $z_{0.001} = -3.0902$. With skewness equal to 0.5, we obtain $\tilde{z}_{CF,0.05} = -1.4980$. A leptokurtic (platokurtic) distribution implies lower (higher) smallest quantiles as shown in Graph V-4a (Graph V-4b). The highest correction for the VaR occurs in the case of a left-skewed and leptokurtic distribution**.** This situation is presented in appendix Graph V-12**.**



a) Platokurtocic distribution: $S = 0$ and $K = 2.5$



b) Leptokurtocic distribution: $S = 0$ and $K = 4$

Graph V-4 - Cornish-Fisher and normal quantiles according to the kurtosis coefficient

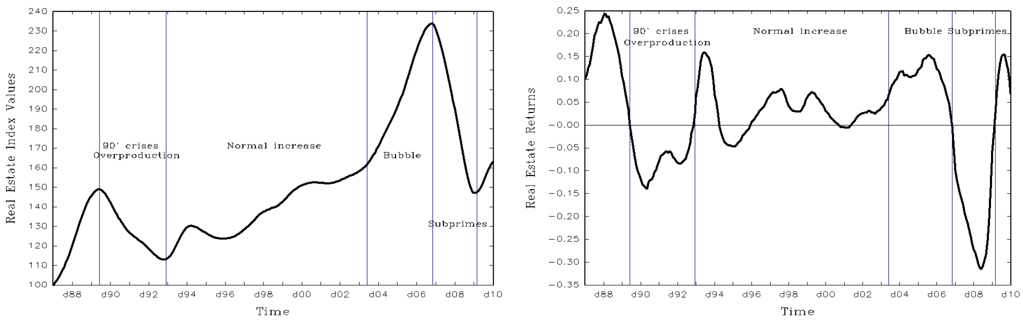
III. Application

We study monthly IPD U.K. real estate capital returns all properties index from January 1988 to December 2010, which is 276 observations.[11] The index is valuation-based. The pitfalls of this specific index (i.e., valuation-based index, smooth, reliability) are discussed in Solvency II calibration paper CEIOPS-SEC-40-10, but are

[11] Solvency II regulation is based on U.K. IPD total return indices spanning 1987 to the end of 2008, totaling 259 monthly returns.

not the purpose of this chapter. Our objective in the following is to apply Cornish-Fisher expansion to compute the VaR of real estate UK capital return and to underline the shortcuts and simplifications made be the regulators who do not consider the asymmetry of returns (more generally the non-normality). The regulator however underlines the shortcuts it was taken but do not provide any solutions or answers: “The figures below demonstrate the standardized distribution (i.e., mean is zero and unit standard deviation) of annual property returns across alternative property market sectors. All distributions of property returns are characterized by long left fat-tails and excess kurtosis signifying disparity from normal distribution. […], albeit the methods do not eliminate the inherent bias. We find […] left tail […] whilst the volatility of the adjusted de-smoothed index is much lower than the volatility of the MSCI developed total return index”. The committee recognizes its conservatism in using the total return index, which inherently assumes rental yield earned is re-invested. We chose the IPD capital return index to avoid the shortcut. The committee recognizes the shortcuts, but in absence of better datasets and because of the low proportion of real estate portfolios for insurers in Europe, it did not improve its work.

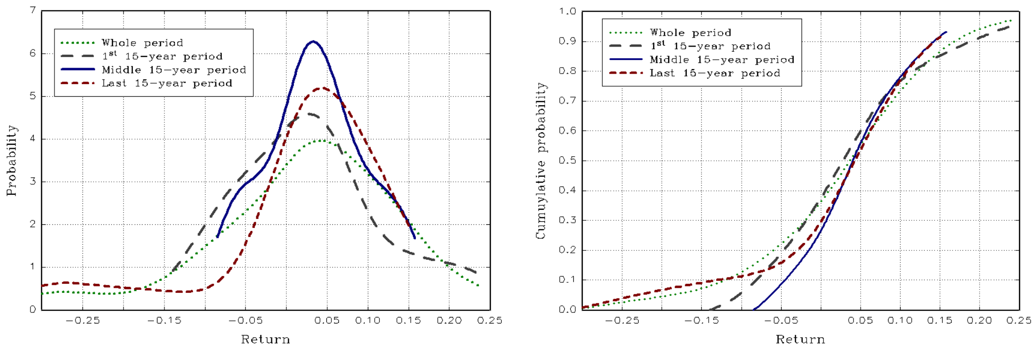
We study the database and determine quantiles and VaR at thresholds of 5%, 1%, 0.5% and 0.1%. The 0.5% threshold is required by solvency II regulation. The values are annualized monthly returns. The index and corresponding returns are presented in Graph V-5, clearly exhibiting both the 1990s overproduction crisis and subprime periods.



Graph V-5 - Real estate index and returns from January 1988 to December 2010

We use bandwidth to obtain VaR evolution. The length of the bandwidth is not determined, neither statistically nor from a real estate viewpoint (e.g., economic analysis, regulators etc.). Statistically, length has to be large enough to enable moment computation. Standard valuation models use a 10-year cash flow period, but it could be longer. For example, the Solvency II standard model for real estate required capital is based on all observations. Since the purpose of our analysis is VaR for non-Gaussian distributions and not the impact of window length, we do not study this point. A 15-year period is used, a compromise between sufficient data and obtaining VaR analysis over time.

Distributions of returns differ across periods as illustrated in Graph V-6. Various curves correspond to the entire period and three 15-year periods: the first 15-year period (January 1988 to December 2002), the middle 15 years (December 1991 to November 2006) and the last 15 years (December 1995 to December 2010).



Graph V-6 - Real estate returns pdf and cdf according to a 15-year period

The middle 15-year period distribution is the more concentrated one, and the high returns from the end of the 1980s and the lower returns from the subprime crises are not considered. The distribution for the last 15-year period is left-skewed, with high negative returns (subprime crises).[12] These negative returns lead to a different analysis of risk when analyzing descriptive statistics presented in Table V-2.

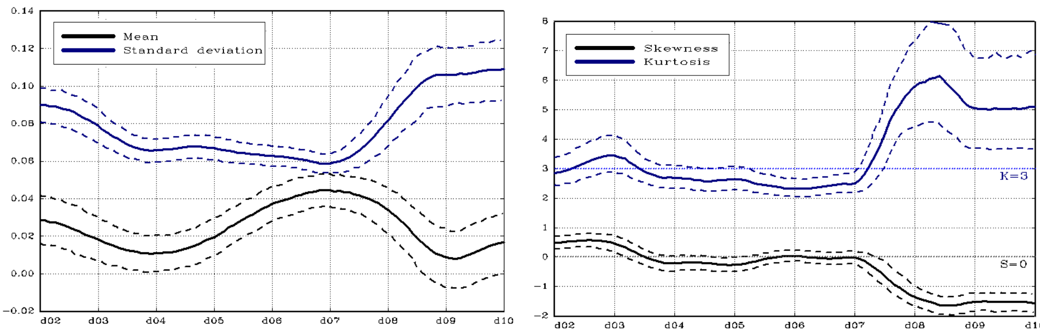
Periods	Mean	S.D.	Min	Max	*S*	*K*	Q1	Q2	Q3
1988-01 / 2010-12	0.0213	0.1132	-0.3154	0.2439	-0.7502	3.7849	-0.0438	0.0305	0.1001
1988-01 / 2002-12	0.0286	0.0903	-0.1396	0.2439	0.4854	2.8450	-0.0407	0.0248	0.0706
1991-12 / 2006-11	0.0357	0.0633	-0.0849	0.1600	0.0363	2.3219	-0.0045	0.0314	0.0732
1996-01 / 2010-12	0.0169	0.1095	-0.3154	0.1543	-1.5666	5.0540	-0.0022	0.0332	0.0770

Table V-2 - Real estate monthly returns pdf and cdf according to the period

[12] This raises a question concerning the window length of the dataset chosen by regulators. A shorter dataset that comes back to today leads to higher skewness and kurtosis coefficients. The subprime and sovereign debt crisis had more important weights.

Mentioned previously, a long dataset is recommended to compute VaR, and we remind here that we decide to compute the following results based on 15 years (i.e., 180 returns). In addition to allowing us to consider more than one cycle, this choice enables obtaining results that are not too erratic, which is the case with small windows. We thus use a 15-year rolling period to compute moments of the returns. Each of the 97 computations contains 180 observations.[13]

Graph V-7 shows the 95% bootstrap confidence interval of the mean, the standard deviation, the skewness coefficients and the kurtosis coefficients. The means and standard deviations are unstable. The returns increase from December 1999 to the subprime crises (during the normal increase and the bubble), and are falling.



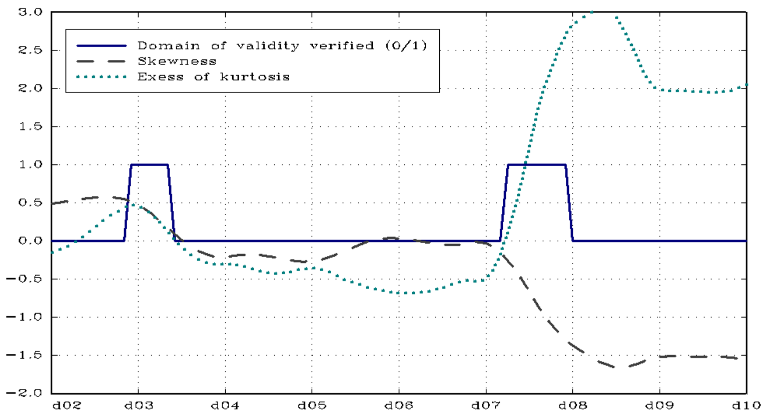
Graph V-7 - mean, standard deviation, skewness and kurtosis according to the 15-year period, and their 95% bootstrap confidence interval

Evolution of the standard deviation appears opposite. Evolutions of S and K are much more dichotomous: i) nearly stable around 0 and 3, the Gaussian values before the subprime crises, ii) highly left-skewed ($S < 0$) and leptokurtic ($K > 3$) afterwards. Until December 2001, the distribution is platokurtic (the kurtosis coefficients are different from 3). From December 2002 to December 2007, the skewness is either null or positive (right-skewed due to high returns as during the bubble period, for example).

Mentioned in Section 2 of this chapter, Cornish-Fisher expansion was subordinated to the monotonic condition. The latter is verified when skewness and kurtosis coefficients belong to the domain of validity, defined as inequality (7). The validity condition is not verified each time kurtosis is lower than 3. Moreover, the combination of skewness and kurtosis coefficients leads to a larger invalidity period. Without Chernozhukov et al. (2010), Cornish-Fisher expansion would be possible only

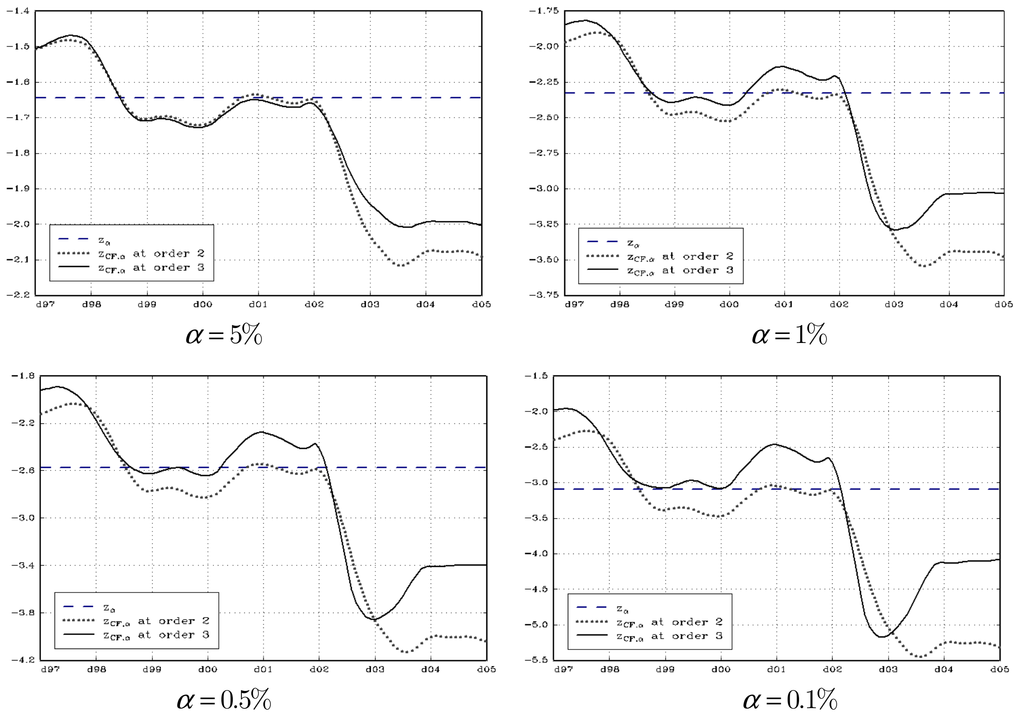
[13] For example, estimation of the distribution in December 2002 was made using returns from January 1988 to December 2002.

in two relatively small periods: from November 2003 to April 2004 and from March 2008 to November 2008 (shown in Graph V-8).



Graph V-8 - Skewness and kurtosis belonging to the domain of validity (D). The represented function equals 0 if $(S,K)\notin D$ and 1 if $(S,K)\in D$ **.**

Given S and K for each 15-year rolling period, we compute Cornish-Fisher correction of Gaussian quantiles. Graph V-9 presents results for the 5%, 1%, 0.5% and 0.1% quantiles of the real estate returns distribution. Approximation is conducted at both orders 2 and 3. The modified quantile at order 3 is, by construction, more precise than the one obtained at order 2. Nonetheless, the latter offers a good approximation of the correction, which must be made for Gaussian quantiles. Whatever the order, the discrepancy between modified quantiles $\tilde{z}_{CF,\alpha}$ and the Gaussian ones z_{α} is high. The dotted lines correspond to Gaussian quantiles. The correction is noticeable during the first months (from December 2002), during the bubble period and largely after the market collapse in December 2007. It is now clear how important this correction is when the threshold of VaR decreases. This correction is especially relevant when considering lower quantiles (e.g., for the 0.5% threshold of Solvency II quantile).

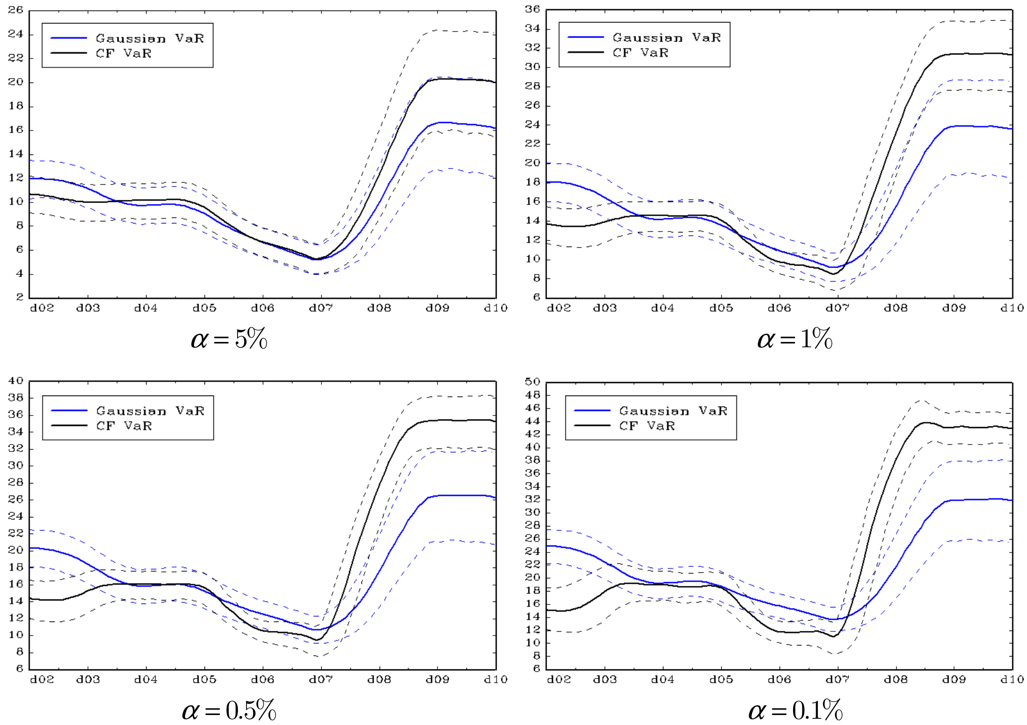


Graph V-9 - 5%, 1%, 0.5% and 0.1% quantiles of the real estate returns distribution according to the 15-year period

The Gaussian quantile is constant whatever the period since S and K are fixed to 0 and 3, respectively. The quantile modification induced by the Cornish-Fisher expansion comes from the fact that the skewness and kurtosis coefficients are not constant over time, as indicated in Graph V-7. This is particularly obvious for the 0.5% and 0.1% quantiles, which are the more interesting quantiles for VaR computation. Thus:

- In December 2002, the modified quantile is less negative than the Gaussian one (i.e., less risky);
- During 2003, the quantile decreases to reach the Gaussian one in the middle of 2004;
- From the middle of 2004 to December 2007 (bubble period), we again obtain less negative quantiles than the Gaussian one for thresholds lower than 5%;
- From 2007 to 2008, a fall in market value leads to higher modified quantiles in absolute value;
- From 2008 to 2009, the modified quantile increases slightly but remains below the Gaussian value (i.e., pronounced for the lowest quantiles).

We now compute VaR given the same threshold with the same 15-year window. The interesting point is to compare results with those obtained from the regulators. Solvency II regulation requires 25% of required capital for real estate investments. Its valuation is based on U.K. all properties total return database with a threshold of 0.5%. Valuation by regulators is close to that obtained with the Gaussian assumption at 0.5% threshold level. However, considering moments of orders higher than 2 leads to higher VaR, and therefore higher required capital. This result demonstrates how essential it is to consider skewness and kurtosis to better estimate real estate VaR.



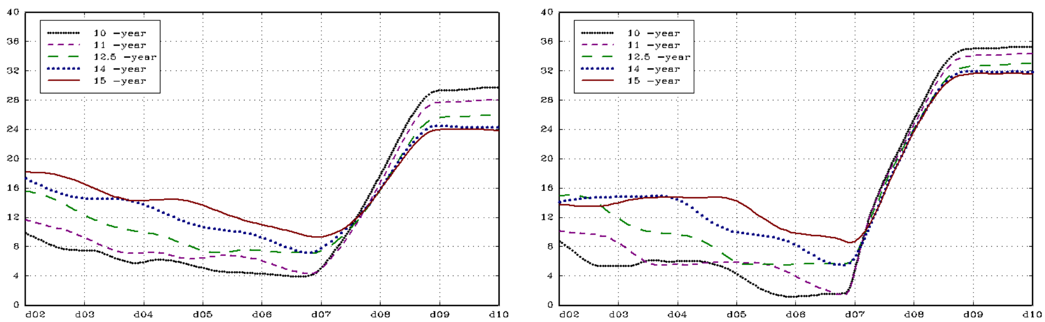
Graph V-10 - 5%, 1%, 0.5% and 0.1% Gaussian and corrected Cornish-Fisher VaR in base 100 according to a 15-year period, and their corresponding 95% bootstrap confidence interval

There are two sources of risk in the confidence interval computation for Gaussian VaR: randomness of mean and variance. Adding to these randomness of the skewness and kurtosis coefficients, we obtain four elements entering the calculus of the corrected VaR. The 95% confidence interval of the corrected VaR is often lower than the Gaussian one. There are more sources of randomness, but since the random variables correlate, the standard deviation of the VaR estimator is lower. Hence, we observe two improvements in VaR:

- The expectation (i.e., value required by regulators) is more reliable; the Gaussian VaR is biased in any case where normality is rejected;
- The precision of expectation is enhanced, which reinforces the previous calculus.

Graph V-10 highlights the corrected VaR (denoted CF VaR on the graph for the Cornish-Fisher VaR) is different from the Gaussian one. The VaR discrepancy is low only for the 2004/2006 to 2007/2012 periods, which was stressed previously concerning quantile analysis (Graph V-9). For lower thresholds (0.5% and 0.1%), there is a difference between the two VaRs during the bubble period; the corrected one is lower. For other periods, correction is high. For example, in December 2010, we obtain about 25% of required capital at the 5% threshold, while the corrected VaR is about 35%, an increase of 40%.

To conclude this analysis, we revisit length of window choice. In Graph V-11, VaR of the 10, 11, 12.5, 14 and 15-year periods are represented simultaneously for the same period. Before the middle of 2008, the longer the window, the higher the VaR. After that date, we observe the opposite effect. Modifications created by the window length are qualitatively the same for the Gaussian and Cornish-Fisher VaRs.



a) Gaussian VaR **b) Cornish-Fisher VaR**

Graph V-11 - Gaussian and Cornish-Fisher corrected VaR for various window lengths

The difference between these two VaRs is stable whatever the windows as highlighted in appendix Graph V-13. This illustrates window length is not relative to the Cornish-Fisher expansion but to the VaR computation, and more generally to distribution estimation. As mentioned previously, regulators could fix the length exogenously.[14]

[14] Many financial areas use a 10-year window.

As all methods proposed for VaR estimation, Cornish-Fisher methods suffer from limitations, and we discuss some limitations briefly.

IV. Conclusion

We propose a VaR computation model that combines Cornish-Fisher expansion and rearrangement procedures. This combination was necessary since Cornish-Fisher expansion suffers from an important pitfall; the cumulative distribution function could be non-monotonic, which makes it inapplicable to many practical situations. The mix of these two methodologies - Cornish-Fisher and rearrangement - allows determining the quantile function and VaR more reliably. This approach bypasses models relying on Gaussian[15] assumptions or other distribution assumptions (e.g., Monte Carlo and parametric method). Our approach does not rely on any distribution assumptions, so we account for non-normality of returns in real estate. In addition, traditional lack of data is overtaken using this approach (historic method,[16] for example, requires a large amount of data).

The objective of the chapter is to compute VaR for a direct commercial real estate investment such as requested by many recent regulations. Results show methods that do not consider skewness and kurtosis to compute VaR lead toworse estimation of risk. In the presence of skewed returns and fat tails, the Gaussian VaR leads to non-adequate capital requirements and under-evaluation. This situation appears especially after the subprime crises.

This article contributes to the existing literature by employing a method based only on the first four statistical moments and has relevance given current regulatory environments. There are good reasons for real estate practitioners, banks and insurers to implement it alongside other models in a real estate context where datasets are modest. When faced with lack of data, no methods provide accurate outputs. Professionals also find here an internal model to determine required capital.

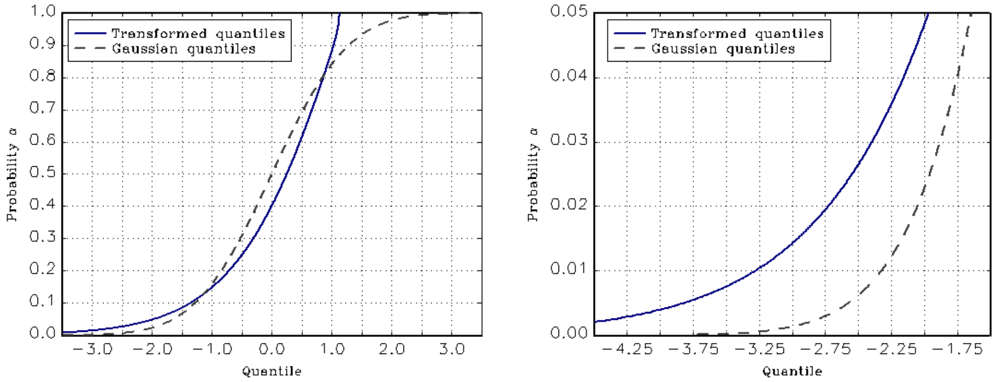
The method is particularly robust for distributions that are non-normal, and therefore applies to hedge funds and private equity sectors, among others. Further studies should be conducted in those two financial fields. VaR is a risk measure that only considers the probability of being below a threshold. It does not consider values

[15] For simplicity, the Gaussian distribution is used, which enables dealing with small datasets.
[16] The traditional, historic VaR is usually not possible or relevant.

below this level or their average. In this sense, researches focusing on measures that consider the entire distribution such as Conditional Value at Risk or expected shortfall should be explored, especially methods relying on Cornish-Fisher expansion. Also, bandwidth size is another issue that requires further study.

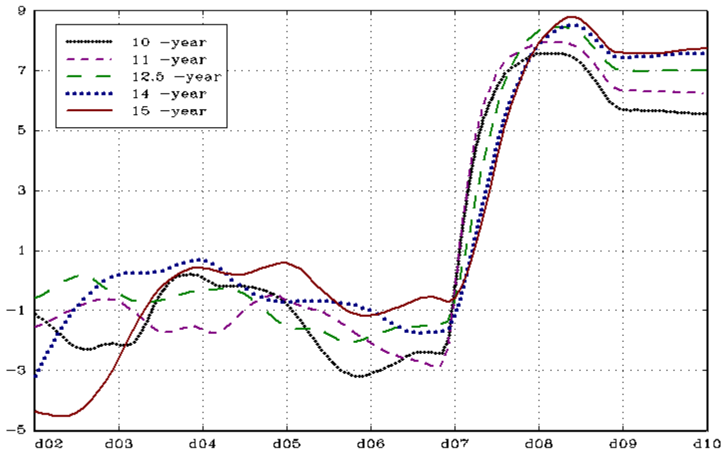
Appendices for chapter 3

Appendix 1: Left-skewed and leptokurtic distribution



Graph V-12 - Cornish-Fisher quantiles, left-skewed and leptokurtic distribution, S=–1.5, K=6

Appendix 2: Comparison of Cornish-Fisher VaR for various window lengths



Graph V-13 - Differences among Cornish-Fisher VaRs for various window lengths

Chapter VI. Value at Risk: a Specific Real Estate Model[1]

In this chapter, we incorporate real estate portfolio specifications in a real estate VaR model. The idea is the following: two portfolios must exhibit different VaRs. However, the traditional methods presented in the introduction of this thesis offer the same VaR whether the portfolio is risky (opportunistic strategy) or secured (core strategy). Even if the approach is acceptable from a risk management viewpoint, results are not ideal. Since real estate investment is difficult to diversify, it is extremely complicated to eradicate specific risk, and therefore specific risk exists in the majority of real estate investor's portfolios. It follows that risk of the entire market does not correspond to risk an investor bears. Our objective is to contribute to real estate risk VaR model by proposing a new model that incorporates all risks borne by real estate investors. The proposed model must be able to discriminate between risky and secured investments.

The approach of this paper derives partially from this thesis's first paper. We improve the VaR computation model by considering characteristics of real estate portfolios. The characteristics considered include lease structures, obsolescence, vacancy costs, vacancy probabilities given an obsolescence rate, rent, market rental values and market prices. We use a method close to the one used in the first paper. We simulate numerous paths of market rental values in a portfolio and prices of each property using bootstrapping. We use bootstrapping instead of Monte Carlo simulation because of the difficulty to determine the distribution of returns reliably;

[1] This study was started by Charles-Olivier Amédée-Manesme who was joined quickly by Fabrice Barthélémy (University of Cergy-Pontoise). The genesis was a presentation at the French Real Estate Research association concerning Solvency II. Numerous people asked how to differentiate risky and non-risky investments given the approach offered by regulators of required capital. We realized no method existed, and so projected to develop one. In 2011, a version of this paper was presented at the European Real Estate Society's annual conference in Eindhoven during a PhD student poster session, and was awarded most commended poster. Stephen Pyhrr (University of Texas) and Nick French (Oxford Brookes University) offered excellent recommendations and comments for improvement of the work. In 2012, an advanced version was presented at the European Real Estate Society's annual conference in a parallel session. Once again, we received excellent comments, especially concerning ways to account for leverage in this kind of model from people at Reading University (Kieran Farelly, among others). The authors wish to thank all the participants of these conferences for the helpful comments.

bootstrapping offers the advantage of not requiring assumptions of the distribution in a series. We consider correlations among variables using the block bootstrap method proposed by Politis et al. (2004, 2009). Then we incorporate specific risk at the property level. We account for lease structure risk following rational behavior; a rational tenant vacates the property at the time of a break option if the property's rent is too high in comparison to market rental values of similar space. We account for length of vacancy following Poisson's law, and link the parameter of the Poisson's law to obsolescence of the property; the higher the obsolescence, the higher the length of vacancy. We determine the obsolescence function of the property (or the way the property becomes vacant in time), and link obsolescence to the vacancy probability. Finally, we integrate the leverage at the portfolio level and include the possibility to breach LTV covenants. The approach considers many criteria that enter property investments and portfolio management. Limitations of this approach reside in the number of parameters estimated and the difficulty of finding reliable databases for the effects we examine.

The primary contribution of this paper lays in assessing VaR by incorporating characteristics of real estate investments. No methods have been developed especially for real estate. Nevertheless, real estate investments harbor many characteristics discussed in the introduction of this thesis that must be considered when computing VaR measures. The results and approach developed in this article provide useful ideas for risk managers and academicians. Research of obsolescence effects on value, tenant behavior, vacancy length and property itself must be conducted to determine the precise effects of all parameters on VaR. The method developed in this paper should be of the interest to professional risk managers willing to construct internal models for capital adequacy purposes. The approach could be adapted - with a few changes - to nearly all asset classes not traded regularly such as art, wine, private equity venture funds, hedge funds etc.

I. Introduction

VaR is one of the most well-known and accepted measures of risk of loss for a portfolio of financial assets. It is recognized as a powerful and efficient risk measure. VaR is appealing because it conveys market risk of the entire portfolio in one metric. Moreover, VaR focuses directly and in local currency terms on a major reason for assessing risk: loss of portfolio value. It offers the advantage of representing investment risk with one number, expressed as a value or percentage. Until recently, VaR was not popular in real estate industry and research, contrary to other financial sectors such as stocks and bonds in which VaR computation is standard. This is due to a lack of data in the real estate industry, particularly commercial real estate data in which transactions are often undisclosed and less frequent. Computing VaR requires reliable and frequent data; this measure is only relevant if a large amount of data is accessible.

Recently, VaR became more popular in real estate in light of recent regulations that require it. Recent[2] regulations such as Bale II, Bale III and Solvency II reinforce bank and insurance capital structures. They require computation of VaR for all investments, including real estate. Practitioners, risk managers and even academicians must consider the VaR of a portfolio before investing or studying. Consideration of return per capital invested is no longer sufficient; return per capital employed (invested and required) is now considered. Thus, VaR is slowly becoming a fundamental risk measure for commercial real estate investments.

Nearly all - if not all – contemporary financial sectors are subject to recent regulations. These regulations are not easily comparable, but they all follow the same objective: assessing the risks borne by financial institutions more accurately with required capital. All of these regulations use either a standard or user-specific, internal model. Calculation methods and assumptions of the standard model are public information: databases, assumptions and methods are available. Standard models propose for each asset class (e.g., stocks, bonds, indirect investments, real estate etc.) standard required capital. Nevertheless, all major institutional investors opt for an internal model. They argue required capital computed with the standard model does not represent the risks they bear. They also argue that the standard model maximizes required capital, and so construct internal valuation models to determine the VaR of their investments.

In real estate, VaR has been the topic of some research. Research of risk focusing on direct real estate or unlisted vehicles is scarce in spite of increasing interest, due partially to the paucity of available data. Either invest in listed real estate and real estate is quoted daily and enough data are available to compute portfolio VaR, or invest in direct real estate and deal with small datasets. This is particularly true in commercial real estate in which institutional investors invest largely and VaR computation is mandatory for capital adequacy. The real estate market is comparable to the private equity market in which indices are constructed

[2] The impetus to use VaR comes from international adoption of the Basel Accord (nearing completion in 2012) in 1999.

from small numbers of transactions. Real estate property indices aggregate real estate market information to provide a representation of underlying real estate performance. However, this is conducted monthly in the best cases, quarterly or semi-annually and sometimes annually, linked largely to sector. The residential field, in which many transactions are observable, generates a monthly index frequently. Commercial real estate faces more difficulties delivering indices with a large periodicity.

Research of VaR in real estate focuses on REITs or securitized real estate, due primarily to data availability. VaR for listed real estate relies on methods for stocks and bonds. Zhou and Anderson (2012) concentrate on extreme risks and the behavior of REITs in abnormal market conditions, suggesting no universal method can be recommended for VaR in listed real estate. They note that estimation of risk for stocks and REITs requires disparate approaches. Cotter and Roll (2011) study REIT behavior over the past 40 years. They particularly highlight the non-normality of REIT returns. They compute VaR for the index following three methods that do not rely on Gaussian assumptions: Efficient Maximization algorithm, the Generalized Pareto Distribution and a GARCH model. The authors highlight that reality is much worse than that depicted by an assumption of normally distributed returns. Liow (2008) uses extreme value theory to assess VaR dynamics of ten major securitized real estate markets. Extreme value theory allows the author to consider the stochastic behavior of the tail. An extreme market assesses risk better than traditional standard deviation measures, and real estate forecasts are more accurate.

Literature focusing on VaR in the context of direct real estate investment (or funds) is sparse and similar to the previous chapter. However, numerous papers concentrate on risk management and assessment in real estate. Booth et al. (2002) study risk measurement and management in the context of real estate portfolios. They argue practical issues force real estate investors to treat it differently than other assets classes. They particularly highlight that direct real estate is a topic for future research. The report is a complete review of the range of risk measures that measure real estate risk, focusing on the difference between symmetric measures such as standard deviation and downside risk measures such as VaR. Their work concentrates on all risk measures appropriate for real estate. They do not conclude with one ideal risk measure, but propose adapting risk measures to needs. Gordon and Wai Kuen Tse (2003) consider VaR a tool for measuring leveraged risk in a real estate portfolio. Debt in a real estate portfolio is a traditional issue in real estate finance. The paper demonstrates VaR offers better assessment of risk. Traditional risk-adjusted measures (e.g., Sharpe ratio, Treynor's and Jensen's alpha) suffer from the leverage paradox. Leverage adds risk along with potential for higher returns per unit of higher risk.

Therefore, the risk/return ratio does not change noticeably and is not an accurate tool for measuring risk inherent in debt. VaR is good tool for leveraged risk. Brown and Young (2011) focus on a new way to measure real estate investment risk. They begin by refuting the assumption of normally distributed returns that flaw forecasts and decisions. The nature of risk and how it should be measured is discussed. Interestingly, VaR is not retained, and expected shortfall is recommended more highly. The authors focus on spectral measures, their final recommendation. Farelly (2012) measures risk of unlisted property funds using a forward-looking approach. Among other results, the author considers moment measurements of orders higher than two (asymmetry considered) using Cornish-Fisher expansion. Following these authors, we focus our paper on the need for better VaR assessments in the direct real estate field.

In real estate investment, no VaR model dominates. Even worse, we do not find a model constructed especially for real estate portfolios or investments. Property investments harbor many characteristics and discrepancies that must be considered when assessing risk. Among them are illiquidity (buying and selling may span months), location (property is immovable), obsolescence (property does not preserve efficiency), lease structures and break clauses (property may be leased long or short-term, possibly with break clauses). VaR has long been applied exclusively to liquid portfolios; that is, portfolios that can reasonably be marketed regularly.

Recent regulations urge methods of computing VaR more reliably. Lack of data is one of the primary motivations to implement traditional VaR methods (e.g., historic, Monte Carlo and parametric) in real estate. However, all methods suffer at least from one limitation; they do not consider the characteristics of the real estate portfolios analyzed. Computing VaR using market data gives the market VaR, not portfolio VaR, VaR. Real estate portfolio characteristics are so different that it is fundamental to consider them in any risk calculation. Few investors argue they replicate the real estate market. They have preferences and invest in one type of risk according to their required return. Some investors invest only in core assets (newly delivered or refurbished properties with a long-term lease); others invest only in risky assets (old buildings to be refurbished or vacant properties). Real estate VaR must therefore consider characteristics and intricacies of a portfolio. This point is particularly obvious for property investments since two investors who invest in the same area but under disparate strategies cannot arrive at the same VaR.[3] Two portfolios that compute VaR

[3] In the same vein, two different portfolios cannot display the same Value at Risk and two identical portfolios with two different strategies cannot display the same VaR.

using the same index or data arrive at the same value if they consider only location and not the characteristics of the investment.

We propose a model that considers characteristics of real estate investments, and the purpose resides in characteristics of the real estate portfolio. We focus on lease structure, state of the properties, possible vacancies, tenant decisions and leverage. We propose a method that considers all characteristics and particularities of a portfolio and incorporate it into a VaR calculation. The objectives are to construct a model that discriminates risky and non-risky investments and obtain a VaR specific to a portfolio.

Before discussing analysis and presentation of the model, we point out assumptions taken in this article. We assume rational investors and players make decisions based on discounted cash flows and returns. We do not consider asset management fees in portfolio management. For clarity, we suppose an optimal world without taxes (Hoesli et al., 2006; Baroni et al., 2007a). We consider neither depreciation nor capital gain tax. We consider leverage in the model and account for the LTV ratio only.[4] Finally, we do not consider arbitrage or investment during a simulation. Therefore, the portfolio is an asset itself, sold entirely at the end of a simulation. These restrictions can be removed easily, but we assume them to keep this article clear and to a reasonable length.

This article chapter is laid out as follows. The next section presents the model. All parameters included in the model are explained and described, and their functions and effects are also discussed. In section 3, we introduce an example based on two portfolios that differ on only a few points. This example illustrates the model well. The parameters and functions presented in the previous sections are estimated or stipulated. Results demonstrate both the impact of each parameter and its relevancy. Section 4 concludes and discusses future research.

> Important note: The model developed below considers all primary factors that influence property values and returns, but more importantly, it accounts for factors that generate risk. We consider parameters that either have a traditional impact on a property's return (i.e., leverage, lease structure, and break options) or are widely regarded as important by investors (i.e., state of property, vacancy costs, probability of vacancy etc.). To do this properly, it would have been necessary to determine both parameters and their

[4] The importance of leverage in the Value at Risk assessment has been studied in Gordon and Wai Kuen Tse (2003).

functions using a database. This was not possible at the time this thesis was written. Databases are scarce and we were not able to obtain them (IPD and PMA refused and Costar is currently studying our idea). For these reasons, we do not determine precisely parameters and their functions, but we do speculate on their functions logically. We hope to obtain a database to determine reliably functions and parameters, and make this part of the thesis publishable.

II. The model

The model we propose was constructed to consider the primary characteristics of real estate investments that influence value and expected returns in the portfolio. These characteristics were chosen because they are points of attention for investors (e.g., length of vacancy). Length of vacancy represents market data, but it is not market data according to property obsolescence, and must be considered in models. At the time a property is purchased, it may be in a good state, but after a few years, the previous state changes (e.g., if spaces are vacated, if a new standard appears or simply because of time etc.), affecting vacancy length.

The model is based on two papers: Baroni et al. (2007a) and Amédée-Manesme et al. (2012). Our proposal relies on these two papers by adopting their ideas: Monte Carlo simulations for terminal and market rental values and option theory for leases. We add numerous factors that affect values or returns of portfolios. Particularly, we are interested in factors that affect portfolio value negatively. Assessing VaR, we assess losses and therefore focus only on factors that affect values or returns.

As already stated, many of the hypotheses offered in this paper are not confirmed through database estimation and cannot rely on extant research. Principally, this comes from poverty of real estate data. Some of the functions stated in this article are purely speculative and are assessed neither statistically nor econometrically. However, the model allows a large degree of freedom and was constructed as a toolbox from which one can chose which functions or factors to incorporate. Estimated functions can be replaced with better ones if and when available.

A. Discounted cash flow

The basis of our proposal is a traditional discounted cash flow model. The present value model is used commonly and accepted widely in real estate and corporate finance. Real estate assets are income-producing investments. We determine price of a portfolio P_0 as the discounted sum of all future free cash flows (FCF; positive or negative) generated by a portfolio, discounted at discount rate k:

$$\forall t, P_0 = \sum_{t=1}^{T} \frac{FCF_t}{\left(1+k\right)^t}$$

In real estate, FCFs are comprised of two parts: recurrent cash flows coming from exploitation of properties (i.e., rent and expenses)[5] and terminal values when properties are sold. Terminal values of a portfolio are only involved at the conclusion of cash flows. It is fundamental to know that by assumption and for simplification, no arbitrage is possible before the end of cash flows and the portfolio itself is sold at the end of a holding period. The two components of the FCFs are:

- Rent and expenses generated by the properties in the portfolio.
  - The FCFs of unique space (or unit) i of property j at time $t \in \left[1; t-1\right]$ gives:

    $$\forall i = 1,\ldots,n,\ \forall t = 1,\ \ldots, T\text{ - }1,\ FCF_t^i = \sum_{i=1}^{n}\left(Rent_t^i - Exp_t^i - Wk_t^i - Cv_t^i\right)$$

    where, as defined by Amédée-Manesme et al. (2012), $Rent_t^i$ is the rent of space i over period t-1 to t, Exp_t^i are common (or recurrent) expenses of space i over period t-1,t, Wk_t^i are capital expenses related to space i over period t-1,t and Cv_t^i are vacancy costs of space i over period t-1,t.
  - FCFs for all spaces in the portfolio comprised of n spaces at time t are:

    $$\forall i = 1,\ldots,n,\ \forall t = 1,\ \ldots, T\text{ - }1$$

    $$FCF_t = \sum_{i=1}^{n}\left(FCF_t^i\right) = \sum_{i=1}^{n}\left(Rent_t^i - Exp_t^i - Wk_t^i - Cv_t^i\right)$$

- The terminal values of all properties.

[5] Expenses can be recurrent or infrequent.

- The terminal value of property j at time t is:
  $\forall j = 1,\ldots,m,\ \forall t = 1,\ \ldots,T\ ,TV_t^j$
- The FCFs generated by all m properties that comprise the portfolio at time T is:
  $\forall j = 1,\ldots,m,\ \forall i = 1,\ldots,n,\ t = T$

$$FCF_T = \sum_{i=1}^{n}\left(Rent_T^i - Exp_T^i - Wk_T^i - Cv_T^i\right) + \sum_{j=1}^{m} TV_T^j$$

This approach determines the value of a portfolio. It is a classic model used by practitioners, particularly appraisers. However, it does not give the VaR of a portfolio. To obtain VaR, we add risk to both parts (i.e., incomes and prices) of the FCFs using quantitative methods.

B. Quantitative methods in discounted cash flow

To incorporate risk in real estate FCFs, we use bootstrapping methods.[6] Generally, bootstrapping is used to estimate properties of an estimator (such as variance) or a dataset. In our case, we do not estimate any properties in the dataset, but increase the size of the dataset to estimate future paths. A shortcoming is the underlying assumption that history repeats. Using bootstrapping, we obtain a distribution of possible values for a portfolio, which allows us to derive VaR.

We apply bootstrapping methods to rent and prices in a portfolio. To consider the correlation between income (rent) and price, we use block bootstrapping[7] methodology as described by Politis et al. (2004, 2009).[8] The approach considers the relationship between rental (rental values) and capital (price) returns, and between rental values themselves and capital returns themselves if many markets are considered.

In the moving-block bootstrap, n-b+1 overlapping blocks of length b are created in the following way: observation 1 to b is be block 1, observation 2 to b+1 is block 2 etc. From these n-b+1 blocks, n/b blocks are drawn randomly with replacement. Aligning these n/b blocks in the order they were picked gives the

[6] Monte Carlo simulations are also possible. Bootstrapping has the advantage that it does not require any assumption on the distribution of the returns. When data are scarce, it can be a benefit.

[7] In the moving block bootstrap, overlapping blocks of predefined lengths are created. Then, blocks of data are selected randomly with replacement (instead of unique data). This bootstrap method works with dependent data, and block lengths can vary.

[8] See also Zoubir and Iskander (2004).

bootstrapped observations. Bootstrapping works with dependent data, however the bootstrapped observations are not stationary by construction. But it is shown that varying block length avoids this problem

For each asset in a portfolio, we use bootstrapping to model both terminal values and all income from all spaces of properties. This action is done at the portfolio level considering correlations among all simulated variables. We repeat this procedure thousands of times and obtain a distribution for all values obtained.

Incorporation of quantitative methods in the discounted cash flow of real estate finance is not new. It has already been proposed by numerous academicians, including Pyhrr (1973) (the first introduction of quantitative methods in real estate), French and Gabrielli (2004, 2005, 2006), Hoesli et al. (2006) and Baroni et al. (2007a, 2007b). Following these authors, we account for uncertainty using quantitative methods (bootstrapping). This allows us to consider risk in income returns and prices. Thus, we consider market risk when computing the value of a portfolio. In theory and according to traditional Markovitz (1952) portfolio theory, specific risk can be diversified so only market risk remains. In real estate, specific risk is difficult (nearly impossible) to diversify, and an incredibly large number of asset is necessary to diversify specific risk. This has long been demonstrated by a number of articles, including Brown (1988, 1991), Barber (1991), Cullen (1991), Byrne and Lee (2001), Callender et al. (2007) and Mitchell (2012). This is why it is fundamental to account for specific risk.

Risk in income returns depends on not only market risk, but also specific risk (or idiosyncratic risk). Market rental value is not income a landlord receives for a unit; the landlord receives rent, and rent rarely equals market rental value. This is due to a lease. Rent is usually indexed over time, and thus increases gradually given a specified index (except in the U.K. where rent is based on an upward-only review of market rental values). Given a market, rent currently paid by a tenant may deviate strongly from market rental value, and thus create risk.[9] This is one of the most important risks borne by a landlord: leasing risk (i.e., when a space becomes vacant). Considering lease structure is fundamental when uncertainty of cash flows is also considered. It allows considering one of the most essential specific risks. To incorporate this risk, we use a model presented in the next sections.

[9] This case occurred often in 2010; many leases contracted in 2007 were outside the market. Between 2007 and 2010, market rental values collapsed in major European cities, given the PMA database.

C. Lease structure

In a parent paper presented in the 3rd chapter of this thesis, Amédée-Manesme et al. (2012) explain how rent is indexed (in most countries, rent is revised regularly based on indices such as inflation and market value) and how lease structures influence cash flows. We focus on the option conceded to a tenant in a lease and the behavior of the tenant facing this option. The possibility to vacate is one of the primary drivers of real estate portfolio return, particularly in the case of a falling market of rental values. We point out some essential features of the model. Rent is generally indexed during the course of the contract (with exception in the U.K.). Contracts are written with clauses in favor of the tenant to terminate the contract before the end of the lease.[10] These options are called break-options (or break-clauses). Facing a break-option (or at exercise time), we assume a rational tenant leaves if the difference between rent currently paid and rental value available on the market for a similar space is not too high. This decision is rational in the sense that the tenant decides to move to a cheaper place if the possibility to do so is available.[11] In a falling market, property can be over-rented (i.e., tenants pay more than the market level). In this case, rent reversions are not always welcome by landlords.[12] Break-clauses represent opportunities for a tenant to spend less, and a rational tenant may exercise this opportunity. Mathematically, the decision to move for a tenant facing a break-option can be expressed as:

$$\frac{MRV_t^i}{Rent_t^i} > 1 + \alpha^i, \alpha > 0$$

where α^i is the moving criteria decision for space i and MRV_t is the market rental value at time t for space i. Quantity α represents bargaining power of the tenant, which can include moving, agency (the cost of a consulting business that helps the company in its decision or the cost of the broker) and human resource costs.[13]

Instead of using bootstrapping for rent and price, we use it for all market rental values and prices (the portfolio is comprised of m properties and n spaces). At relevant dates, simulated market rental values are compared to rent paid, and the most rational decision is taken. If a space is let too much above current market value at the

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[10] Options embedded in a lease are asymmetric because only the tenant can exercise the option.

[11] Moving or agency costs may be considered.

[12] Owners or asset managers may be tempted to negotiate, grant free rent, upgrade or offer financial incentives to maintain a tenant. These are not considered in the model, but could be an improvement of this approach and of Amédée-Manesme et al.'s (2012) recommendations.

[13] The company may face defection of some workers due to location changes.

time of a break option, the tenant vacates and the landlord faces a void. We now have a model that considers both vacancy and a tenant breaking a lease before the end of the contract. A difference here lays in the way rent is estimated.

We take two hypotheses that can be relaxed in the model, but that are necessary for clarity of presentation. After a period of vacancy (or after a break-option is exercised), a property is relet at market rental value. At the end of a lease when the option is a dual option, both tenant and landlord can make a decision; the tenant and the landlord decide to renew the contract at market rental value.[14] This follows the idea of rational behavior (the landlord wants to increase revenue and avoid vacancy, and the tenant avoids moving and agency costs).

Therefore, this paper introduces the possibility of vacancy. It is essential in any real estate cash flow model to consider leases, particularly lease structures and vacancies incurred by those structures. Disparities that occur due to a lease structure have a colossal impact on future cash flows and on risk of the investment. When calculating VaR, the risk profile of the portfolio, with or without leasing or property risk, is important. A VaR computed for a real estate portfolio that faces many break-options in the near future (and therefore many leasing risks) must be higher than a VaR computed for a core portfolio without leasing risks, even if both investors - core and opportunist - have sufficient assets to diversify specific risk. A VaR valuation model must incorporate leasing and property risks. This is why we incorporate lease structure risk in the VaR model. Now we consider two issues arising from vacancy: length of vacancy and cost of vacancy. Vacancy costs influence property value according to the length a unit or property remains vacant.

D. Length and cost of vacancy

Vacancy is one of the greater risks investors bear. A vacant space generates costs and obsolesces more quickly. We face two issues: how long a space remains vacant and how much vacancy costs. We propose a solution for each and suggest incorporating them into our VaR model.

Following Amédée-Manesme et al. (2012), we model vacancy length using Poison's law. Therefore, length of vacancy follows:

$$p(k) = P(X = k) = e^{-\lambda} \frac{\lambda_i^k}{k!}$$

[14] This decision is financially rational for both tenant and landlord. The tenant minimizes expenses (no moving costs, broker fees etc.) and the landlord maximizes revenues because vacancy is avoided.

with λ a positive real number equal to the average vacancy length and k the number of occurrences of the event. Average vacancy length in a market is available easily from databases or brokers. These data can be segmented by sub-markets and sometimes sub-sub-markets (e.g., new versus second-hand premises).

Vacancy costs represent spending estimated to be paid due to vacancy. This arises because in most rental contracts, a tenant pays current expenses and only large capital expenses are paid by the landlord. Estimation of vacancy costs is difficult because expenses in cases of vacancy vary widely by property and location. Technical characteristics, tenants, services and location of a property influence the types of charges faced during void periods. Costs faced by a landlord during vacancy come from various sources. Landlords may have to spend marketing expenses or broker fees. Some local taxes usually charged to a tenant must be paid by a landlord during vacancies.[15] A landlord may also need to keep heating or air-conditioning to keep the spaces in a good state. In addition, landlords often must maintain services for other tenants on the property.[16] These costs are part of common expenses charged proportionally to the sizes let, and therefore the landlord must pay the parts arising from vacant spaces.

Estimating cost of vacancy is complicated, requiring much information and many parameters. However, an experienced real estate asset manager can determine vacancy costs of a real estate asset. It only remains to count these costs in cash flows when a space is vacant.

To model them, we propose a percentage of estimated market rental value. The percentage must be estimated using a database. In absence of a database, a percentage can be fixed to an arbitrary value or estimated through experience or valuation. Therefore, vacancy cost at time t for space i is:

$$Cv_t^i = 1_{vacant} \times \xi(t,i) \times MRV_t^i$$

with $\xi(t,i)$ a function representing vacancy charges for asset i at time t. 1_{vacant} is the indicatory function and Cv_t^i is the amount of vacancy cost defined in II.A of this chapter.

Thus, each time a space becomes vacant, vacancy cost and length of vacancy are considered. However, many factors influence vacancy length and the probability of vacancy. One primary factor is the obsolescence rate of the property (or the state of the property). In commercial real estate, after location—which largely defines rent—the state of the property influences a tenant's decision strongly.

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[15] Properties such as towers and service offices face particular charges.

[16] This is often the case in a half-let property in which a landlord faces higher vacancy costs than with an empty property. A landlord pays half of reception-desk costs, half of security and half of canteen subscriptions.

E. Obsolescence (depreciation)

There exists some confusion surrounding the terms depreciation and obsolescence. The two are often used interchangeably, as we do. However, it is clear depreciation has several recognized and major impacts on property that obsolescence does not have, particularly when considering accounting and taxation. Only one discernible academic development of a body of knowledge and theory defines and classifies these concepts: Baum (1991). Nevertheless, much literature exists in accounting that concentrates on techniques used in the application of depreciation to profit and loss accounts. In addition, RICS Asset Valuation Standards Committee guidelines offer partial definitions for depreciation and obsolescence for property sectors. For our purposes, we consider obsolescence a cause and depreciation an effect. Since our objective is not to define or redefine terms, we use one comprehensible concept: obsolescence.

Obsolescence represents a decline in competitiveness, usefulness and/or value of a property. Obsolescence occurs due to availability of alternatives that perform better, cheaper or both, or changes in user preferences, requirements or style. Property obsolescence affects value in two ways: physical deterioration and functional decline. Salways (1986) and Baum (1991, 1994)[17] describe categories of building obsolescence, including aesthetics (outdated appearance), functional (technological progress that changes an occupier's requirements), legal (safety or environmental) and social (demands for a secure environment). Obsolescence is a highly complex topic that is difficult to evaluate reliably and credibly. We do not model obsolescence rate or propose an obsolescence function[18] because those could be topics of an entire thesis, but we do consider obsolescence in the VaR model. In the application section, we assume linear decline of property competitiveness.

A property does not preserve efficiency over time due to normal use, changes and environment. To account for obsolescence, we introduce function $\theta(t,j)$, assuming property efficiency decreases over time. By reciprocity, obsolescence of a property increases during its life. Therefore:

$$\forall t, \forall j, \frac{d\theta(t,j)}{dt} > 0$$

[17] See also: Barras and Clarck (1996).

[18] An obsolescence model can be found in Baum (1991).

Obsolescence is represented by a percentage of the total value of the property. Note that property preserves residual value even when obsolete; this value can come from location or attractiveness of the shape of the building (e.g., historic façade). Obsolescence thus has a strong, direct influence on value, but also indirectly on asset management. An obsolete property faces difficulties attracting and retaining tenants.

F. Probability of vacancy according to obsolescence

The state of a property is a fundamental part of its value, and essential to remain attractiveness to a tenant. Maintaining a tenant in an old or obsolete asset can be difficult. Leasing an old or obsolete property is more difficult than leasing a new one, even for a cheaper price.

Formalizing and quantifying risk of vacancy is essential to obtain a good understanding of real estate risk. We consider the probability of vacancy increases with obsolescence. We incorporate probability of vacancy in moving criteria decision α. This criterion may have a huge impact on a tenant's decision, positive or negative (in cases of extremely obsolete properties), and therefore influence a decision taken by a tenant strongly when facing a break option. Therefore:

$$\forall t, \frac{d\theta(t,j)}{dt} > 0 \Rightarrow \forall t, \frac{d\alpha_t^{i,j}(\theta,t)}{d\theta(t,j)} < 0$$

where θ is the obsolescence function depending on time t and property j, a function that increases over time, and $\alpha_t^{i,j}$ is the moving criteria for space i of property j at time t. The moving criterion is a function that influences the probability of vacancy according to the state of the property. Since the more a property is obsolete, lower are the moving decision criteria, and the probability of vacancy increases. Ideally, the moving criteria decision is estimated using a complete database. However, this was not possible, and a credible function is assumed during application. We thus include the probability of vacancy more rapidly given property obsolescence. To influence the decision of the current tenant, the obsolescence rate has a strong influence on the length a property remains vacant.

G. Length of vacancy according to obsolescence

The state of a property influences attractiveness strongly. Therefore, an obsolete or out-of-date property remains vacant longer than a newer one,[19] even if rent is lower. Average vacancy length in our model is market average (or sub-market average), and therefore, some properties remain vacant longer than others do. We hypothesize that obsolescence influences expected length of vacancy positively.[20] Mathematically, this is expressed as:

$$\forall t, \forall i, \frac{d\lambda\left(\theta,t,i\right)}{d\theta\left(t,j\right)} > 0$$

where $\lambda\left(\theta,t,i\right)$ is the average length of vacancy function for space i at time t. Again, a database estimates this function better, but this was not possible for this paper. We modify average length of vacancy by incorporating a portion that varies according to obsolescence rate.

H. Debt

Debt is the last element influencing risk of a real estate portfolio. Debt affects returns but also investment risk in light of extreme events such as those estimated in a VaR model. Leveraged investments offer better returns in cases of favorable situations. In cases of bad situations, investors may not recover anything, and can lose all invested capital. Leverage adds risk in addition to potential higher returns. However, common risk-adjustment measures have serious limitations when applied to leveraged portfolios. The Sharpe ratio and other risk-adjustment measures (e.g., Treynor's or Jensen's alpha) does not allow differentiation between risky and non-risky investments because, by definition, it results in comparable levels. Thus, risk/return ratio does not change appreciably with leverage. VaR offers better consideration of leveraged risk. In our model, debt is not considered at the portfolio level, but it is considered at the property level. This is often the case in real estate finance because property is viewed as a covenant of a loan and landlords do not often accept cross-collateral to remain free

[19] An obsolete property located in an area where all properties are obsolete does not suffer from the obsolescence rate. A refurbished property in an area where all properties are obsolete either attracts tenants more easily as the best property in a market or is too expensive or beautiful for tenants in the market.
[20] This is true assuming tenants are rational and assuming spaces are priced at market rental values.

from decisions (cross-collateral blocks management decisions). Covenants are the terms or clauses in a loan agreement.

Given limited benefits, lenders must focus on limiting their downside. As a result, a secured lender underwrites loans based on many ratios. The four most common are loan-to-value (LTV), debt-service coverage, interest coverage and fixed-charge. While these ratios are important, covenants are far more important and contentious. To limit disadvantages of a risk profile, lenders demand as many loan covenants (or securities) as possible.

We concentrate[21] solely on the LTV, and more precisely on the covenants attached to the ratio. LTV represents the ratio of the first mortgage lien as a percentage of the total appraised value of real property. For example, if an investor borrows 50 M€ to purchase a property worth 100M€ at relative valuation, the LTV ratio is 50/100 or 50%. Risk of default is always at the forefront of lending decisions, and likelihood of a lender absorbing a loss in a foreclosure process increases as the amount of equity decreases. Therefore, as the LTV ratio of a loan increases, the qualification guidelines for loans become increasingly severe. In addition, cost of debt (margin) depends on LTV; low LTV ratios often accompany low debt margins.

The agreement generally allows the LTV to vary from a percentage (δ) of the initial LTV due to market movements (and appraisals). We assume loan conditions are breached if property value is lower than the minimum accepted by (or negotiated in) a loan agreement.

For property j at time t, if $\forall t$, $\forall j$, $LTV_t^j > LTV_0^j + \delta$ the loan is breached. δ represents permitted variation of the initial LTV. Since breach of LTV covenant is a default, the lender demands immediate repayment of the loan[22] or preempted of the property.

We consider leverage risk in our VaR model. As soon as the simulated value of the property moves too far from initial LTV (LTV_t^j), the value of the property falls to 0 (because it is preempted), and no further income is generated by the property. Other loan risks, especially those related to income, are more complicated to model.

[21] Other ratios are also fundamental for lenders, but differ from market to market, and are subject to negotiation.

[22] If the lender decides not to call in the loan, the lender usually requires the loan to be renegotiated in terms less favorable to the borrower, including higher interest rates and substantial fees. Some loans require a borrower to provide a deposit as further security in the event of a breach of the LTV ratio, thus increasing the financial burden of the borrower. There are indications that lenders are increasingly willing to renegotiate a loan instead of calling it in, given they want to limit bad debts. It is in their interest to agree to a solution that works for both parties.

Particularly, opportunistic loans often do not cover income ratios, but pay debt service with equity. This is why we do not account for it.

III. Application

A. Data and presentation of the portfolios

To illustrate the relevancy of our model, we consider two portfolios that differ only by investment type. One is a core (secured) portfolio and the other is an opportunistic (risky) portfolio. The portfolios are comprised of 10 properties invested in Europe in two markets: France (Central Paris, outside Golden Triangle) and Germany (Central Munich). The properties of the portfolios are located in the same market and sub-market. They differ only in size, leasing and state of properties. Five properties have been acquired in each city. The French part of the portfolio holds 7 units (two properties have two spaces to let, the others are mono-tenant properties), and the German part holds 6 units (one property divided into two spaces).[23] More information on strategy includes:

- The core portfolio is invested only in new developments or recently refurbished properties with long-term leases. Its objective is to distribute recurrent cash flows to an investor. It does not consider leasing or property risk. Leverage is limited and equal to 30%. This investor has long-term investment objectives (i.e., greater than 8 years).
- The opportunist investor acquires only empty (with lease risk) new developments or old, obsolete properties with short-term leases or vacancies. This investor bears obsolescence and leasing risks, and the objective is to realize capital gains. The portfolio is leveraged at 60%, and the investment objective is short (i.e., fewer than 6 years).

The two portfolios have the same initial acquisition price of 1,000 M€, but initial yields are different. They were acquired on the same date. Our model requires certain inputs. In addition, some parameters must be estimated if data are available. Required inputs include:

[23] We assume a space cannot be divided. In practice, units are divided frequently as a function of tenant wishes.

i. Price of each property;
ii. Rent for all units in the portfolios (rent can be 0);
iii. Market rental values for each unit;
iv. Leases of all let spaces and expected leases in case of breaks;
v. Indexation of each unit;
vi. Evolution of the two markets in terms of market rental values and prices;
vii. Average vacancy length for each sub-market (only two sub-markets are considered: Paris and Munich);
viii. Cost of capital
ix. Vacancy cost of each unit;
x. State of each property and how this state is expected to evolve (obsolescence function of each property);
xi. Moving criteria decision for each market and its evolution (probability of vacancy);
xii. Length of vacancy function (dependant on state of the properties);
xiii. General information such as portfolio LTV, debt margin etc.

Inputs i to v: Inputs i. to v. are available in the annual report of each fund. In Table VI-1, we show information concerning these inputs. For simplicity, each property has a price of 100 M€ for a portfolio value of 1,000 M€. All new leases are expected to be contracted according to 6/9-year contracts in France and 5/10-year contracts in Germany. We assume all leases are indexed at 2% per year (in practice, discrepancies[24] can occur; Germany and France do not use the same index or a similar indexation). Market rental values, passing rent at the time of acquisition and the lease structure of each space are displayed in Table VI-1.

[24] Rent is indexed annually at 2%. In practice, indexation can be negotiated according to many rules. French leases are indexed annually on a specific index (Cost of Construction Index), and German leases are indexed each time cumulated national inflation reaches 10%.

Property #	**City**	**Unit #**	**Market rental values (M€)**		**Rents (M€)**		**Current lease structure**		**Lease structure in case of break**
			Core	**Opportu**	**Core**	**Opportu**	**Core**	**Opportun**	
Property 1	Paris	Unit 1	3.1	4.1	3.2	4.1	9	4	6/9
		Unit 2	3.1	4.6	3.2	-	7	-	
Property 2	Paris	Unit 3	3	4.3	3.5	4.5	9/12	5	
		Unit 4	3	5	3	-	9	-	
Property 3	Paris	Unit 5	6.3	9.5	6.3	-	8	-	
Property 4	Paris	Unit 6	6	8.2	6	8.3	9	2/5	
Property 5	Paris	Unit 7	6.4	8.9	6.3	-	6/9	-	
Property 6	Munich	Unit 8	6	7	6.1	-	10	-	5/10
Property 7	Munich	Unit 9	6.1	8.9	6.3	9	10	4	
Property 8	Munich	Unit 10	6.1	9.7	6.1	-	8	-	
Property 9	Munich	Unit 11	6	7.9	6.5	-	7	-	
Property 10	Munich	Unit 12	3.4	4.5	3.1	4	9	6	
		Unit 13	3.1	4.8	3.3	-	7	-	
Total / Average			**61.6 M€**	**87.4 M€**	**62.9 M€**	**29.9 M€**	**8.36 years**	**1.5 years**	
Financial Vacancy					**0%**	**66%**			

Table VI-1 - Market rental values, rent and lease structures of units in the portfolio.

The acquisition price of each property is the same, but their initial yields are not. The core portfolio is acquired for a net initial yield of 6.2%[25] and the opportunistic portfolio for 8.7%. The riskier portfolio is thus bought for a higher yield, representing higher risk borne by the investor (258 bp). The core portfolio is fully let and has no financial vacancy. The opportunistic portfolio has 8 vacant spaces, representing 66% financial vacancy. Both portfolios are slightly over-let (passing rent > MRV). The let spaces of the core purchaser are slightly over-let by 2.5%. The opportunist purchaser acquired the portfolio over-let by just 1% (on the let part). Differences between passing rent and market rental values are globally low. These differences can be explained by indexation: some rent was indexed positively through years when market rental values were descending.[26]

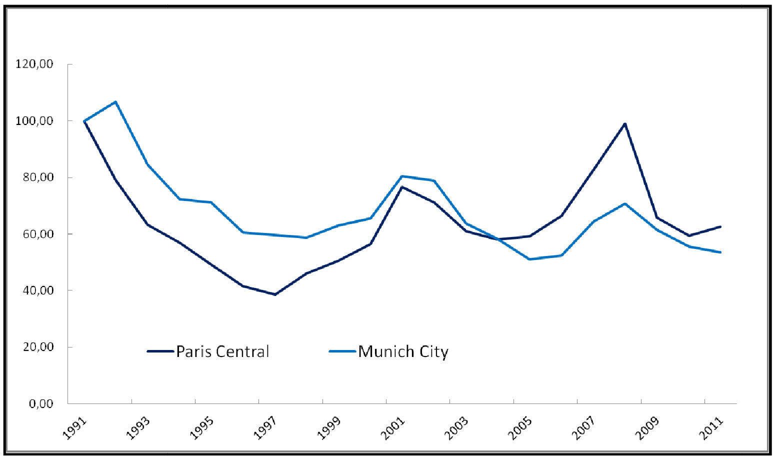
The two portfolios have existed a number of years, and spaces have thus been marketed and let at various dates and periods. Some have been running for many years and are close to the end of the contract; others have just been signed and will produce income for many years. Leases that have just been signed are signed at market rental value, given the assumption of rationality. Other leases exhibit differences from market

[25] Based on market rental values and to make numbers comparable, net initial yield = market rental values/portfolio value.

[26] This occurred often after the 2007 financial crisis when rent was indexed positively and market rental values collapsed.

rental value; some are over-let and some are under-let. Future break options of all leases are shown in Table VI-1. Note that the core portfolio has more than 1 year of secured cash flows and the opportunistic portfolio has only 1.5 years.

Input vi: Evolution of the two markets is public information displayed by data providers or brokers. We rely on the Property Market Analysis (PMA[27]) database (1991-2010), which covers the prime market.[28] This database is not best, but it is one we were able to access and where city data were available. We use market rental values and price evolution given the PMA database. The two evolutions are shown in Graph VI-1 and Graph VI-2. Obviously, market evolution does not depend on strategy. Strategy serves investors interested in various risk/return frameworks. Our objective is to construct a VaR model that differentiates the two investments.

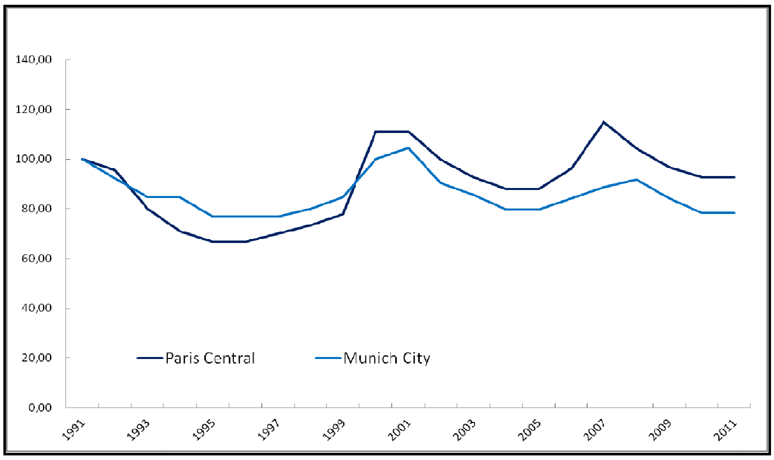


Graph VI-1 - Market capital growth (1991 to 100)

Source: PMA Database

[27] PMA is a data provider specializing in real estate markets.

[28] A prime market is characterized by location. Prime assets are located in the best areas of a city, let under a long-term lease to grade-A tenants (i.e., rating > AA).



Graph VI-2 - Market rental growth (1991 to 100)
Source: PMA Database

As shown in Graph VI-1 and Graph VI-2, the series considers two cycles: the first from 1991 to 2000 and the second from 2001 to 2007. The two cities correlate as do rental growth and capital growth. The lowest correlation is between capital returns of Munich and rental growth of Paris. The correlation matrix is shown in Table VI-2.

	Paris Central (capital returns)	**Munich City (capital returns)**	**Paris Central (rental growth)**	**Munich City (rental growth)**
Paris Central (capital returns)	100%			
Munich City (capital returns)	56%	100%		
Paris Central (rental growth)	74%	19%	100%	
Munich City (rental growth)	68%	62%	73%	100%

Table VI-2 - Correlation matrix market rental values and prices in Munich and Paris
Source: PMA Database

<u>Input vii:</u> Average vacancy lengths are also market data. For Paris, we obtain an average vacancy length of 2 years, and 2.5 years for Munich.

<u>Input viii:</u> Cost of capital in absence of debt (unleveraged fund) is equal to 5.5%. This number represents a premium of 150bp on average 10-year Sovereign debt from 2001 to 2010 (4% for France and 3.9% for Germany). The risk premium used here is based on the assumption that property should have a risk premium over government

bonds. Factors influencing the premium include volatility, liquidity and transparency.[29] Following the PMA database recommendation, we use 150bp for both markets.

> As already addressed during this chapter and more generally during this chapter, we were unable to find a reliable database with state of properties, and therefore unable to determine functions and inputs required in such databases such as obsolescence, moving criteria decision and length of vacancy according to obsolescence rate. To illustrate the relevancy of our approach and the impact of parameters in the model, we make reasonable assumptions about the shapes of these functions. This does not change the relevancy of the model because these assumptions can be changed and relaxed across markets or countries and the model was constructed like a toolbox in which an analyst can decide which parameters to include.

<u>Input ix:</u> Vacancy costs can change across properties and locations. Without accurate data for each property, it is difficult to estimate cost of vacancy per unit. For this reason, we chose a common rule for all spaces in the properties. Cost of vacancy equals 15% of market rental value whatever the state or number of tenants. Thus, the cost of vacancy for space i is:

$$Cv_t^i = 1_{vacant} \times 0.15 \times MRV_t^i$$

<u>Input x:</u> The initial states of properties are known, usually available in the last appraisal report. However, the way states evolve is unknown. Therefore, it is the obsolescence function (and not the obsolescence rate itself) that must be estimated. Given lack of data concerning real estate markets, we once again assume the obsolescence function. We use a simple rule: obsolescence rate of a property increases gradually over 30 years. After 30 years, a property is obsolete (without works or capital expenses). Note that obsolescence is a loss in the real, existing use value in comparison to the market. Our obsolescence assumption is not calculated using land value; an obsolete asset preserves remaining market value due to land and letting values.

We define 6 states of properties presented in Table VI-3. Following our assumption of a 30-year life before obsolescence, a property remains in each state 5 years. For a 10-year discounted cash flow model, the state of the property changes

[29] See Jones Lang LaSalle Transparency index.

twice. Assumption of linear erosion (excepting land) is probably incorrect. Reality lays between accelerated (quicker at the beginning) or reduced erosions (quicker at the end).

Possible state of a property
- 1: New or completely refurbished - Labelled
- 2: very Good state
- 3: Good condition
- 4: In need of some attention
- 5: Old
- 6: To be refurbished / obsolete

Table VI-3 - Possible state of properties

Input xi: Probability of vacancy considered in light of a moving criteria decision is a difficult parameter to assess. It depends on each tenant, and can be modified given business characteristics. We assume a moving criteria decision that changes gradually from one market state to another. A newly developed property has a moving criteria decision equal to 1.25 and an obsolete one equal to 0.75 as shown in Table VI-4.

Possible state of a property	**Moving criteria decision**
- 1: New or completely refurbished - Labelled	1.25
- 2: very Good state	1.15
- 3: Good condition	1.05
- 4: In need of some attention	0.95
- 5: Old	0.85
- 6: To be refurbished / obsolete	0.75

Table VI-4 - Moving criteria decision associated with state of the property

In the absence of property states, the moving criteria decision is fixed arbitrarily to 1.05.

Input xii: Length of vacancy is a function of state of the property; the more a property is obsolete, the longer it remains vacant.

Possible state of a property	Average vacancy length Munich	Average vacancy length Paris
- 1: New or completely refurbished - Labelled	2.00	1.50
- 2: very Good state	2.50	2.00
- 3: Good condition	3.00	2.50
- 4: In need of some attention	3.50	3.00
- 5: Old	4.00	3.50
- 6: To be refurbished / obsolete	4.50	4.00

Table VI-5 - Average vacancy length associated with state of the property

Input xii: In our model, leverage is considered explicitly, driven by the LTV ratio. LTV is debt compared to appraised value of a property. Given market movements, LTV ratio moves over time, and debt agreements allow movement around the initial LTV (δ). In this application, the core portfolio has an LTV ratio of 30% and the opportunistic one has an LTV of 60%. The swap rate operates independent of strategy and is fixed to 3.5% (the average of the past 10 years of swap rates from 2000 to 2010). However, the margins are different from one strategy to another, with a margin of 130bp for the core portfolio and 250bp for the opportunistic one. Thus, cost of debt equals 4.8% for the core portfolio and 6% for the opportunistic portfolio.

Now we come to negotiated LTV. The core portfolio has low initial debt and can negotiate greater movements in the market. Thus, $\delta = 0.4$ (LTV = 70%). The opportunistic portfolio has lower power of negotiation and can only negotiate low movements in the market: $\delta = 0.25$ (a total LTV of 85%). Since market movement might be temporary, we assume the lender exercises its option only if the covenant is breached during two consecutive periods (one period equals one year). After two periods of LTV covenant breach, the property is preempted by the lender and the value falls to 0. In addition, it no longer generates rent.

According to debt, the cost of capital changes following the weighted average cost of capital. We consider a cost of equity (required return) of 8.5% and 20% for the core and the opportunistic portfolio, respectively.

We thus have all inputs, the parameters and functions necessary to compute VaR, and implement the model for the two portfolios. Table VI-6 shows a comparative summary of the two portfolios. The opportunistic properties exhibit a higher rate of vacancy and a stronger obsolescence rate.

	Portfolio # 1	Portfolio # 2
Strategy	**Core**	**Opportunistic**
Nb properties	10 - 5 in France - 5 in Germany	
Nb spaces	13 - 7 in France - 6 in Germany	
Portfolio Value	1,000	
Market rental value	60.3	80.0
Passing rent	60.0	29.5
Vacancy rate	0%	63%
Nb vacant space	0	8
Weighted average lease length	8.36 years	1.5 years
Lease structure in case of BO	- France: 6/9 - Germany: 5/10	
Average state of property	1.3	4.3
WACC	5.5%	
LTV	0.3	0.6
Swap rate	350 bp	350 bp
Margin	130 bp	250 bp
Covenant breach	70%	85%
Cost of equity	8.5%	20.0%
WACC	7.3%	11.6%

Table VI-6 - Comparisons of the portfolios

B. Implementation of the model and results

Using a traditional VaR model, we begin implementation by computing VaR for the two portfolios. The computation is conducted using thresholds of 5.0%, 1.0% and 0.5%. We then progressively add each source of risk and observe the effects on VaR.

- **Historic approach (total returns[30])**

[30] Total returns represent combined returns from rent and prices, similar to a reinvested dividend index.

The historic method involves taking empirical profit and loss histories and ordering them, assuming history repeats. To compute historic VaR for a portfolio, we consider yearly total returns of the two countries and compute historic total returns for the two portfolios by weighting (50% for each) the cities' indices. Due to the small number of available data, we were only able to compute 5% VaR. We obtained an historic VaR at 5% of 18% ($VaR_{0.05}^{historical} = 0.18$). The historic[31] method offers the same result whatever the risk or portfolio.

Portfolio total return		**Cities total return**	
Years	Munich + Paris	Munich	Paris
1990	16%	4%	29%
1991	-1%	-13%	10%
1992	-15%	-14%	-17%
1993	-7%	-4%	-10%
1994	-3%	-9%	3%
1995	-10%	-10%	-10%
1996	1%	-2%	3%
1997	15%	27%	3%
1998	14%	16%	12%
1999	13%	18%	8%
2000	34%	41%	26%
2001	0%	-3%	2%
2002	-11%	-8%	-15%
2003	-2%	0%	-4%
2004	-1%	7%	-8%
2005	12%	16%	7%
2006	27%	27%	27%
2007	18%	22%	13%
2008	-18%	-28%	-9%
2009	-5%	-5%	-6%
2010	17%	26%	9%

Table VI-7 - Portfolios and cities' returns

- **Parametric approach (total returns)**

The parametric[32] approach is based on an assumption concerning a statistical distribution (normal, log-normal etc.) from which data are drawn. In this paper, we

[31] To make this approach statistically reliable, ensure a sufficient number of observations is available and that they are representative of all possible states for the portfolio.

[32] The distribution may not reflect all possible market states accurately and may under or overestimate risk. With only 20 data points, choice of distribution is a huge dilemma.

assume a normal distribution of returns. Thus, portfolio returns follow $N(0.04;0.14^2)$. If we define z_α, the standardized Gaussian quantile at threshold α, which verifies $F_Z(z_\alpha)=\alpha$, the corresponding quantile q_α of the distribution is:

$$\forall\alpha\in\left]0,1\right[,\ q_\alpha=\mu+z_\alpha\sigma$$

The VaR at threshold α for an investment is:

$$\forall\alpha\in\left]0,F_Z\left(\tfrac{-\mu}{\sigma}\right)\right[,\ VaR_\alpha(I_0)=-q_\alpha\times I_0$$

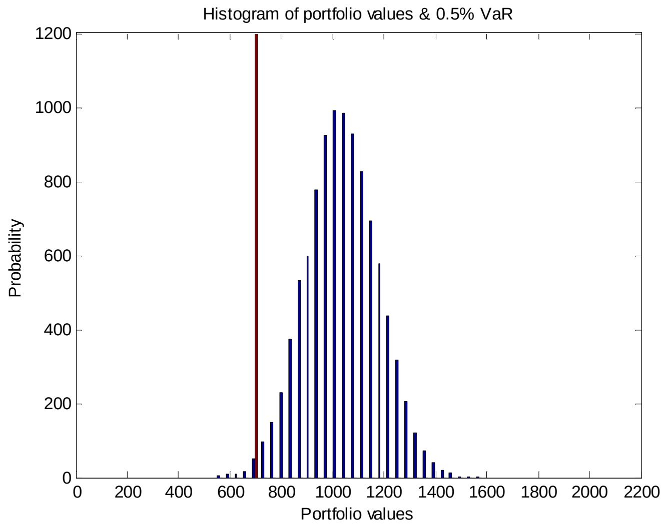
Results are presented in percentage (in absolute term) in Table VI-8. Once again, this method does not allow discriminating risky and core investments.

α	$z\alpha$	$q\alpha$	VaR (%)
5%	-1.645	-0.188	19%
1%	-2.326	-0.285	28%
0,5%	-2.576	-0.320	32%

Table VI-8 - Value at Risk using the parametric approach

- **Monte Carlo simulation (total returns)**

Monte Carlo methods rely on repeated random generation from a probability distribution of inputs that are then used to compute model results. The approach requires an assumption on the distribution of returns. In this paper, we again assume a normal distribution. Thus, portfolio returns follow $N(0.04;0.14^2)$. We simulate 10,000 paths over one year and compute VaR for the portfolio. Results are shown in Table VI-9.



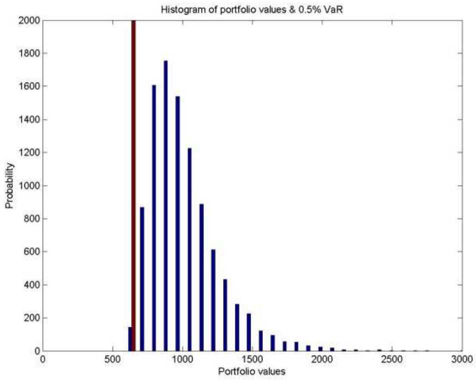
Graph VI-3 – Distribution of Monte Carlo results for the VaR computation

α	VaR (%)	VaR_α
5%	18.7	812.57
1%	27.5	724.51
0.5%	30.7	692.31

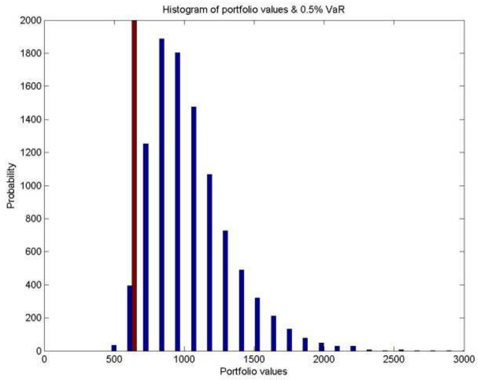
Table VI-9 – VaR with Monte Carlo simulations

- **Bootstrapping (market rental values + properties prices) + leases structures (length of vacancy unaffected by obsolescence)**

Now we present the model we propose. We add each risk one at a time that we suggest considering. We only present a few cases, but all possible cases and their results are presented in appendix 1. We only consider leases structure when the length of vacancy is unaffected by obsolescence rate of the property. Here, vacancy length equals, on average, the market mean.



a. Core ($VaR_{0.5\%}$ = 34.7%)



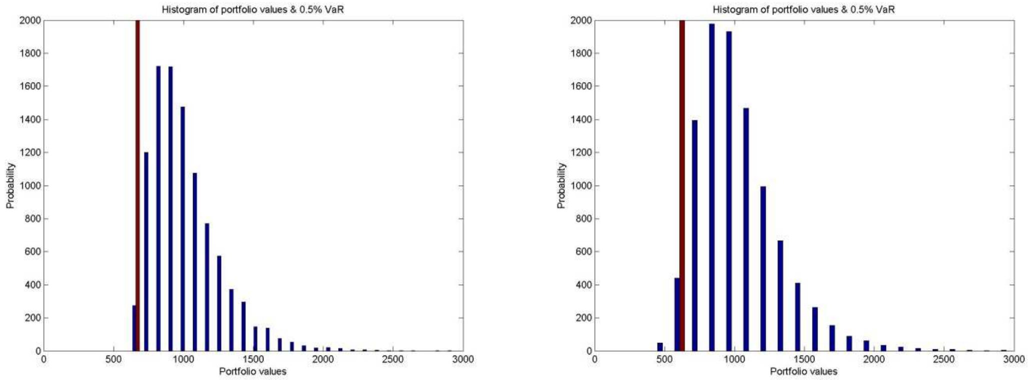
b. Opportunist ($VaR_{0.5\%}$ = 41.8%)

Graph VI-4 - Approach combining bootstrapping for market rental values, and property prices and lease structure

This model allows differentiating between strategies in computation of VaR, obtained by considering characteristics of real estate investments. Note the difference between the two strategies: 7% in absolute terms and 17% in nominal difference. This demonstrates the relevancy of our approach, which considers the specifications that drive differences between the strategies.

- **Bootstrapping (market rental values + properties prices) + leases structures (length of vacancy not affected by obsolescence) + cost of vacancy**

Now we add cost of vacancy (15% of MRV). We consider costs incurred by the vacancy of a unit. Obviously, this parameter influences the opportunistic portfolio profoundly. None of the units in the core portfolio was vacant, and the assumption of rationality led us to consider a negotiation to market rent at the end of the lease, thereby avoiding vacancy. The parameter has thus a small impact on the core portfolio. The risky portfolio is affected by vacancy costs. It increases VaR by 4% (7.6% in absolute terms) in comparison with the case where this cost is not considered. This result justifies considering this factor.

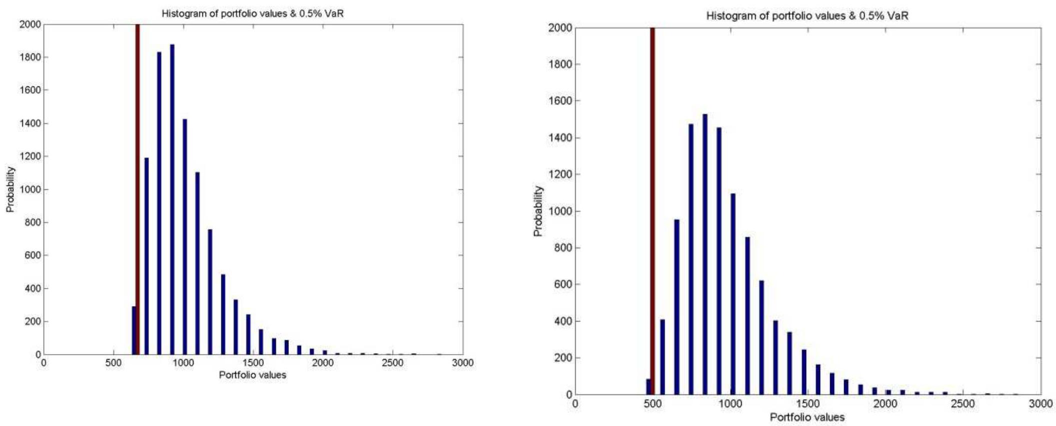


a. Core ($VaR_{0.5\%}$ = 34.5%) **b. Opportunist ($VaR_{0.5\%}$ = 45.0%)**

Graph VI-5 - Approach combining bootstrapping for market rental values, and properties prices, lease structure and cost of vacancy

- **Bootstrapping (market rental values + properties prices) + leases structures + cost of vacancy + length of vacancy (affected by obsolescence)**

We now consider obsolescence rate of a property in length of vacancy. A recent property remains vacant for a shorter period and an obsolete one for a longer period. This has nearly no effect on the core portfolio, which has no (or almost no) vacant units during simulation. The opportunistic portfolio, however, is affected slightly by this parameter; its VaR at 0.5% threshold increases by 4% in comparison to the value obtained when cost of vacancy and lease structure are considered.



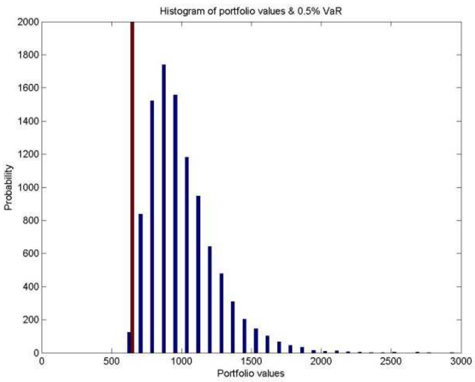
a. Core ($VaR_{0.5\%}$ = 34.8%) **b. Opportunist ($VaR_{0.5\%}$ = 48.7%)**

Graph VI-6 - Approach combining bootstrapping for market rental values, and properties prices, lease structure, cost of vacancy and length of vacancy

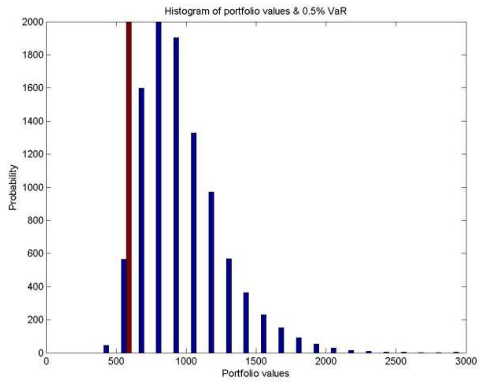
Note that the impact of obsolescence on length of vacancy is also strong. Therefore, the effects of obsolescence on length of vacancy must be considered when computing real estate VaR.

- **Bootstrapping (market rental values + properties prices) + leases structures + cost of vacancy + length of vacancy (affected by obsolescence) + probability of vacancy (affected by obsolescence)**

The probability of being vacant in this application will not show a strong effect. The core portfolio is comprised primarily of recent properties let under long-term leases, and therefore the expected impact of this parameter is low. For the opportunistic portfolio, the impact will again be weak because the units are already vacant and thus will be relet under longer leases.



a. Core ($VaR_{0.5\%}$ = 34.9%)



b. Opportunist ($VaR_{0.5\%}$ = 48.9%)

Graph VI-7 - Approach combining bootstrapping for market rental values, and properties prices and lease structures considering obsolescence in the cost of vacancy and length of vacancy

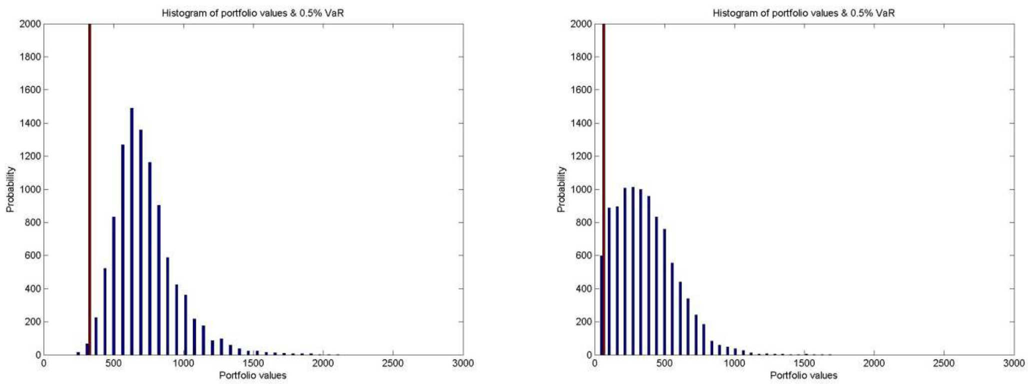
Note that probability of being vacant is relatively low in this application. Either the few number of leases that present break-options[33] (already vacant for the opportunistic portfolio and long-term leases for the core portfolio) or the few number of properties that are old in the core portfolio explain this result.

- **Bootstrapping (market rental values + properties prices) + leases structures + cost of vacancy + length of**

[33] The hypothesis of renewal at the end of the lease is also one of the reason.

vacancy (affected by obsolescence) + probability of vacancy (affected by obsolescence) + Debt level

Finally, we add debt to the model. Level and cost of debt is higher for the opportunistic portfolio. In addition, the opportunistic portfolio has a lower freedom for the LTV covenant. This is why the expected impact on VaR is higher for the risky portfolio. This portfolio may be affected more by cost of debt because all vacant spaces are affected negatively by debt during the time they remain vacant. Note that the distribution changes when debt is considered; the distribution of the opportunistic portfolio becomes particularly left-skewed.



a. Core ($VaR_{0.5\%}$ = 54.1%) **b. Opportunist ($VaR_{0.5\%}$ = 94.4%)**

Graph VI-8 - Approach combining bootstrapping for market rental values, and properties prices, lease structure and debt considering obsolescence in the cost of vacancy and length of vacancy

Again, the model enables us to discriminate between core and opportunistic investments. We note a strong increase in VaR incurred by debt. For the opportunistic portfolio, this leads to a VaR of about 90%. This result is challenging for supporters of high debt, but we justify the result with the following: the period includes two strong declines in property value (mid-1990s and beginning of the century), and lenders in these periods faced covenant breaches. Lenders avoided these issues with linear amortization of a loan or other covenants (e.g., deposits or other properties). These parameters can be added easily to the model, but are not presented here to keep the model understandable.

IV. Limitations and hurdles

There are obvious limitations to our approach we would like to underline. The purpose of this section is to provide ideas for improvement of the model rather than explaining method inadequacies.

Using bootstrapping methods, our model implicitly assumes the past repeats. Obviously, this is false. However, in absence of relevant databases, it is nearly impossible to determine the distributions of returns reliably. We suggest bootstrap methods to avoid assumptions concerning returns distributions. This assumption can be easily overridden with any other distribution according to market data. Another issue arising from bootstrapping is that past data does not necessarily account for improvements in an area or future developments. External events are frequent in real estate such as political decisions, changes to local taxes and public transportation improvements. This can be corrected case by case with an additional factor representing future attractiveness (or ugliness) of a market or sub-market.

Incentive and negotiation strategies are not considered correctly in this model. Landlords generally maintain tenants on a property by offering free rent and financial incentives. Our model, which considers rational behavior, does not replicate the market. The bargaining power of landlords and tenants can be considered in threshold level α (the moving criteria decision may include a utility function that considers tenant preferences), but even with that threshold, decisions are not necessarily rational and other external or emotional factors enter the decision process.

Capital expenses are not considered in the model. We do not find any rational rule that allows incorporating capital expenses. In fact, large capital expenses occur only when a property is vacant and the property is in need of improvements. Therefore, we do not include capital expenses in the VaR model. Nevertheless, we are conscious that opportunistic strategies in real estate finance focus primarily on capital expenses and repositioning strategies; the objective is to buy an obsolete property, refurbish it and relet it at higher rent in a long term-lease (core). This limitation is a challenge for future research that concentrates on opportunistic strategies.

The way debt is considered is questionable. The debt-service coverage and interest coverage ratios are not considered in our model. However, we know these two ratios are important to lenders. How ratios based on income can be included in our model is unclear. It is difficult to include them rationally with a rule common to all strategies. For example, opportunistic portfolios do not generate sufficient cash flows when debt is issued, and therefore covenants are breached immediately. Core strategies do not leverage many investments, and usually do not have trouble with these ratios. This is why we concentrate solely on LTV. Once again, this can be the subject of future research. Debt and leveraged risk have not been subjects of sufficient research in real estate finance, but recent attention of European insurers on commercial real estate debt (following Solvency II regulation) will probably enhance research in the field.

We emphasize again lack of data in a real estate context. Writing this article (and this thesis), our primary difficulty was lack of data for many aspects of the model. Functions proposed in this study were not estimated, and thus rely on logic alone. The model remains nevertheless reliable. The general idea is consistent and functions certainly vary across markets. Publication of this paper is uncertain without a credible database that allows justifying functions assumed. The model is trustworthy, but requires estimated functions.

V. Conclusion

The purpose of this article is to contribute to VaR method development in real estate finance. Traditional VaR models suffer from a problem in real estate investment: they rely on market indices or data, and therefore provide the same result regardless of portfolio. Yet real estate investments are characterized by heterogeneity of assets and thus market indices generally do not reflect portfolio risk. Contrary to traditional approaches, our approach allows discriminating strategies from VaR results. This is useful to both academicians who want to increase real estate understanding and professionals who need to consider all risks they bear when investing in real estate, and who must now compute VaR for capital requirements (principally given Solvency II regulations).

The method incorporates real estate characteristics in the VaR model. Above all, they are required due to difficulties of diversifying idiosyncratic risk in real estate finance. This is why we consider leasing, vacancy, cost of vacancy, obsolescence and leverage risks. Besides these, we also account for market risk by including it in property prices and in market rental values using bootstrapping; we construct a VaR model that accounts for market and specific risk. The method is applied to two real estate portfolios, and we illustrate the relevancy of the model and show that the model can be used to compute VaR for a portfolio. We especially demonstrate that our approach returns disparate VaRs for various portfolios.

The model opens doors to numerous future studies. Debt, strategy and rational behavior should be subjects in future research. Obsolescence of a property and its effects should be particularly questioned, primarily in the context of environmental impact.

To the best of our knowledge, this is the first attempt to consider real estate characteristics when computing VaR for real estate finance. The model is particularly relevant because it is the first that permits different VaRs for various portfolios even if the portfolios differ only with respect to leasing strategy. However, limitations remain and future research is planned. Rationality of players in a market should be examined, especially using a utility function.

Appendix of chapter 4

Core strategy	Opportunistic strategy	Cost of vacancy	Probability of vacancy	Length of vacancy	Debt Level	0.5% VaR	1% VaR	5% VaR
X	-	-	-	-	0%	34,7%	32,9%	27,3%
X	-	-	-	-	30%	54,2%	49,3%	37,1%
X	-	-	-	X	0%	34,6%	33,0%	27,4%
X	-	-	-	X	30%	54,1%	48,2%	36,4%
X	-	-	X	-	0%	34,8%	33,0%	27,5%
X	-	-	X	-	30%	54,1%	49,1%	38,0%
X	-	-	X	X	0%	34,3%	33,0%	27,5%
X	-	-	X	X	30%	54,2%	50,8%	37,4%
X	-	X	-	-	0%	34,5%	33,0%	27,4%
X	-	X	-	-	30%	54,2%	50,0%	37,6%
X	-	X	-	X	0%	34,8%	33,2%	27,2%
X	-	X	-	X	30%	54,2%	50,8%	37,1%
X	-	X	X	-	0%	34,2%	33,0%	27,0%
X	-	X	X	-	30%	54,2%	50,8%	37,8%
X	-	X	X	X	0%	34,9%	32,7%	27,1%
X	-	X	X	X	30%	54,1%	48,3%	36,4%
-	X	-	-	-	0%	41,8%	39,4%	30,5%
-	X	-	-	-	60%	92,5%	91,3%	84,8%
-	X	-	-	X	0%	45,0%	43,0%	35,6%
-	X	-	-	X	60%	93,1%	92,4%	87,3%
-	X	-	X	-	0%	42,6%	39,7%	31,2%
-	X	-	X	-	60%	93,1%	92,5%	84,7%
-	X	-	X	X	0%	45,7%	43,2%	35,6%
-	X	-	X	X	60%	92,6%	92,0%	87,0%
-	X	X	-	-	0%	45,0%	41,9%	32,8%
-	X	X	-	-	60%	94,4%	93,5%	87,0%
-	X	X	-	X	0%	48,7%	46,3%	37,7%
-	X	X	-	X	60%	94,4%	93,8%	89,7%
-	X	X	X	-	0%	44,2%	42,2%	32,9%
-	X	X	X	-	60%	94,3%	93,1%	86,4%
-	X	X	X	X	0%	48,9%	47,0%	38,3%
-	X	X	X	X	60%	94,4%	93,4%	89,0%

Table VI-10 – All possible scenarios of the two portfolios and their computed VaRs

GENERAL CONCLUSION

This thesis has addressed the question of risk in real estate investments: the asset's uniqueness and specifications in returns distributions. This was motivated by the fact that the marginal reduction of idiosyncratic risk decreases rapidly after 10 properties;[34] it is why the thesis concentrate on specific risk in real estate. The original question of the thesis: how to account for specific risk in real estate is answered in several ways. The thesis constructs new models that account for specifications of real estate investments, and proposes new ways to estimate VaR that incorporate characteristics of real estate distributions. This work is significant in the sense that contrary to many recent studies of real estate, which concentrate primarily on empirical databases, this dissertation proposes new approaches that both improve traditional approaches and are useable by practitioners. Originality lays in part in consideration of real estate characteristics and in a practical approach to proposed methods. This work should be interesting to both practitioners and academicians.

Motivations for writing this thesis are highlighted in the general introduction. Motivation primarily came from lack of academic research in the real estate field, particularly from continental Europe, and because real estate accounts for half of the world's wealth. The thesis improves comprehension of general real estate and specific risk. It particularly develops the literature review to offer a global representation of the current state of research in the area. This allowed to present findings and facts on which this dissertation is constructed. One of the primary conclusions of the literature review is this observation: real estate exhibits many features missing in other assets and that make this area distinct. One peculiarity caught attention: specific risk cannot be diversified properly without a very large number of assets. The purpose was to incorporate characteristics of real estate assets in portfolio management and in risk valuation and management. The thesis considers property features (e.g., lease structures, heterogeneity, liquidity, obsolescence etc.) and characteristics of real estate returns distributions (i.e., peaked and left-skewed). This was done across four papers

[34] Most literature suggests the number of assets required in a real estate portfolio is much larger than that held in most institutional portfolios.

that treat real estate portfolio management and risk valuation with particular emphasis on lease structures and VaR measures, respectively.

In terms of methods, the thesis either constructs new models that account for asset specifications or applies recently developed mathematical models to those specifications. The thesis combines quantitative methods and option theory to account for leasing risk and to obtain estimation of cash flow risk across a classic discounted cash flow model. Other factors can be included in the model (e.g., bargaining power). This first model is constructed and complicated to determine the optimal holding period of a real estate portfolio and compute real estate VaR. Second the research combines Cornish-Fisher expansion and rearrangement procedure to compute the quantile function of the returns distribution. This allows fulfilling the thesis's objectives: accounting for abnormal distribution risk and an asset's features.

Findings of this thesis are twofold. First, still today, the specific risk is not evaluated appropriately in real estate, and in this sense; the current work can improve comprehension of idiosyncratic property risk, especially leasing risk. Second, the two new approaches that allow better valuation of VaR in real estate improve the practice of risk management and capital requirements. Paper 3 demonstrates that accounting for non-normality of returns augments required capital in comparison to a standard model. Again, this highlights a need for more research. Globally, this thesis suggests real estate risk is not estimated correctly.

This thesis implies numerous practical applications, partially because it was written during a French doctoral contract (*CIFRE*) that allows theses in the private sector (BNP Paribas Real Estate) and therefore allows exploration of real business issues. The model developed in the first paper, which accounts for leasing risk, can be used in practice for multiple purposes such as optimal holding period (presented throughout paper 2), debt level and risk (Amédée-Manesme and Dupuy, 2010), inflation hedging and more generally portfolio management. Risk can be estimated more reliably. Similarly, the two VaR models can be used by regulators to improve internal models or by professionals for risk valuation and capital requirement adequacy. In addition, use of VaR for portfolio allocation has become popular. This thesis can therefore have a huge impact on risk assessment in real estate businesses, particularly if regulators understand and adopt this approach.

This work has - of course - some limitations, highlighted throughout the thesis (even if it is not the case in classical papers written for publication). None are irremediable. This conclusion does not list all limitations, but does mention the most salient. The main limitations are the rationality assumption and lack of data, which are discussed thoroughly in this thesis.

This thesis does not claim to be exhaustive, but presents a comprehensive treatment of real estate risk. It combines basic quantitative and qualitative methods, but above all, it opens a way to many future studies. Below is a list of some questions to address (and answer) in the future. First, the term structure of rent according to break-options can be questioned. Break-options have a huge impact on a landlord's cash flows, but also on the way rent can be estimated. Second, implied volatility of real estate has not been given much attention even if volatility, unfortunately, is a common risk measure in finance. Implied volatility of real estate can be estimated using the break-options contained in leases by comparing rent level and incentives conceded to tenants with rent and incentives when no breaks are included in the lease. The method implicitly links the premium paid for an option and the rent and incentives negotiated. Third is tenant behavior. This study assumes rational behavior of players in a real estate market, but this assumption can be questioned. Many decisions in real estate are not driven purely by rational factors, but also by sentiment, and using utility functions allows considering irrational player behaviors. Fourth is liquidity. In real estate, time on the market plays an important role in decisions, and can be a bad decision when marketing an asset. This point was recently the subject of some papers, particularly in the United States where liquidity of residential real estate assets was questioned after the subprime crisis. There exists a huge line of future research in this area, with liquidity being nowadays a recurring issue for investors and academicians. Fifth are risk measures. Regulators chose VaR to manage financial institutional risk, but this measure faces many criticisms, and further research can be envisioned, particularly of risk measures such as expected shortfall and fractals. Sixth are the distributions of real estate returns and the possible use of co-skewness techniques in order to better understand the real estate returns (see for example: Carmichael and Coën, 2011). Last is portfolio management. Portfolio allocation and management are covered largely in literature (specialized or not in real estate). Nevertheless, something is lacking in literature: managing real estate risk exposures using a risk budgeting approach. Over the last fifty years, mean-variance optimization was used widely to manage asset portfolios and build strategic asset allocation. Today, given required capital computations, the risk budgeting approach may become a popular method of allocating portfolios.

Developments made in this thesis demonstrate a need for more real estate research. The approaches incorporate specific features found in real estate portfolios and risk management. The significance of these articles relies primarily on idiosyncratic risk. Success depends on dissemination, and thus on publication. The importance and relevancy of this work have been identified. An early version of the fourth paper presented in the PhD session at the ERES[35] conference in Eindhoven in 2011 was awarded *most commended poster* by the ERES committee. The third paper was awarded *most commended paper* during the PhD session in Edinburgh in 2012 at the ERES conference.

*
*   *

There is no doubt real estate practice and knowledge will grow, even designing models, software, methods and enhanced databases for future generations. More research will be devoted to make these data and tools easier to use in industry discussions. “Learn from yesterday, live for today, hope for tomorrow. The important thing is not to stop questioning,” said Albert Einstein. I hope to continue reflecting on and conducting research, no matter how hard it is as long as I never stop questioning.

[35] European Real Estate Society.

Conferences and publications

2010:

- American Real Estate Society annual conference in Naples, FL
  Paper presentation: Combining Monte Carlo Simulations and Options to Manage Risk of Real Estate Portfolio; with M. Baroni, F. Barthélémy and E. Dupuy;
- European Real Estate Society annual conference in Milan, Italy
  Paper presentation: Combining Monte Carlo Simulations and Options to Manage Risk of Real Estate Portfolio; with M. Baroni, F. Barthélémy and E. Dupuy;
  Research presentation: Real Estate Portfolio Optimal Debt Structure: A pan-European Approach; with E. Dupuy.

2011:

- American Real Estate Society annual conference in Seattle, WA
  Research Presentation: Long-term inflation hedging properties of direct real estate investment: a methodology to study inflation's protection given the lease structure and the indexation uses; with M. Baroni;
- European Real Estate Society annual conference in Eindhoven, Netherlands
  Poster presentation: A model to compute Value at Risk for direct real estate investments;
  Received ERES doctoral price: *Best poster presentation.*

2012:

- The American Urban Real Estate and Urban Economics Association in Chigago, IL
  Paper presentation: Combining Monte Carlo Simulations and Options to Manage Risk of Real Estate Portfolio; with M. Baroni, F. Barthélémy and E. Dupuy;
- American Real Estate Society annual conference in St Pete, FL
  Paper presentation PhD session: Value at Risk: a specific real estate model;
  Paper presentation: Optimal holding period for real estate portfolio; with M. Baroni and F. Barthélémy;
- Association Française de Finance annual conference in Strasbourg, France
  Paper presentation: Combining Monte Carlo Simulations and Options to Manage Risk of Real Estate Portfolio. With M. Baroni, F. Barthélémy and E. Dupuy;
- European Real Estate Society annual conference in Edinburgh, Scotland
  Paper presentation: Value at Risk: a specific real estate model.
  Paper presentation PhD session: Cornish-Fisher for real estate Value at Risk. With F. Barthélémy;
  Received ERES doctoral price: *highly commended paper;*

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Summary

The academic contribution of this thesis is in providing an estimate of the risk for managing commercial real estate investment. Property investment is subject to numerous specificities including location, liquidity, investment size or obsolescence, requiring active management. These particularities make traditional approaches to risk measurement difficult to apply. We present our work in the form of four papers on real estate portfolios and risk management. This research is built on extant literature, and relies on previous research, examining first the implication of the option of the tenant to vacate embedded in leases and the implication of this for portfolio value, risk and management. The thesis then concentrates on valuation of Value at Risk measurements through two new approaches developed especially for real estate.

In the first paper, we consider options to vacate embedded in continental Europe leases in order to better assess commercial real estate portfolio value and risk, conducted through Monte Carlo simulations and options theory. The second paper considers the optimal holding period of a real estate portfolio when options to break the lease are considered. It relies directly from the first article, which has already treated this kind of option. The third paper proposes a model to determine the Value at Risk of commercial real estate investments, considering non-normality of real estate returns. This is conducted through a Cornish-Fisher expansion and rearrangement procedure. In the fourth paper, we present a model developed for real estate Value at Risk valuation. This model accounts for the most important parameters and specifications influencing property risk and returns.

Résumé

Cette thèse contribue à la recherche académique en immobilier par l'apport d'une estimation du risque pour la gestion d'immobilier commercial d'investissement. L'investissement immobilier compte de nombreuses particularités parmi lesquelles la localisation, la liquidité, la taille d'investissement ou l'obsolescence et requiert une gestion active. Ces spécificités rendent les approches traditionnelles de mesure du risque difficile à appliquer. Ce travail de recherche se présente sous la forme de quatre articles académiques traitant de la gestion de portefeuille et du risque en immobilier. Ce travail est construit sur la littérature académique existante et s'appuie sur les publications antérieures. Il s'attache d'abord à analyser les options de départ des locataires contenues dans les baux commerciaux en Europe continental et en étudie les impacts sur la valeur, la gestion et le risque des portefeuilles. Ensuite, la thèse étudie l'évaluation d'un outil de mesure du risque en finance, la Value at Risk au travers de deux approches innovantes spécialement développées pour l'immobilier.

Dans le premier article, nous prenons en considérations les options de départ des locataires inclus dans les baux en Europe continental pour mieux évaluer la valeur et le risque d'un portefeuille de biens d'immobilier commercial. Ceci est obtenu par l'utilisation simultanée de simulations de Monte Carlo et de la théorie des options. Le second article traite de la durée de détention optimale d'un portefeuille immobilier lorsque sont prises en compte les options contenues dans les baux. Le troisième article s'intéresse à la Value at Risk et propose un modèle qui tient compte de la non-normalité des rendements en immobilier. Ceci est obtenu par la combinaison de l'utilisation du développement de Cornish-Fisher et de procédures de réarrangement. Enfin dans un dernier article, nous présentons un modèle spécialement développé pour le calcul de Value at Risk en immobilier. Ce modèle présente l'avantage de prendre en compte les spécificités de l'immobilier et les paramètres qui ont une forte influence sur la valeur des actifs.